

Homework for Wednesday, February 13th

1. Let $\mathbf{S} = \{x\}$. What are the elements in the Free semigroup on \mathbf{S} .

2. Let $\mathbf{S} = \{a, b\}$, and let $\mathbf{R} = \{aa = \phi, bb = \phi, ab = ba\}$. Let \mathbf{V} be the finitely-generated semigroup on \mathbf{S} with relations \mathbf{R} . Do the following:

What are the elements of \mathbf{V} ?

Write a multiplication table for \mathbf{V} . That is, draw a chart with each element in \mathbf{V} along the top and along the lefthand side and make a grid just like you were multiplying integers, but now we are using the binary operation of concatenation.

Take an index card and put a mark on both sides of one corner. This is simply to keep track of what position the index card is in. Let F_v denote "flipping" the card over vertically, let F_h denote "flipping" the card over horizontally. Let R denote "rotating" the card one half turn. Let N denote "do nothing." Since we can do these operations one after another, we can think of "multiplying" one action by another by first doing one then doing the other. For example, since flipping the card horizontally and **then** flipping the card vertically together put the card in the same position as it would be if you had rotated the card one half turn (check it), we would say that $F_h * F_v = R$. Complete the multiplication table for these four actions.

Are there any similarities between these two tables?

3. Let $\mathbf{S} = \{a\}$. Let $\mathbf{R} = \{aaaaa = \phi\}$. Let \mathbf{U} be the finitely generated semigroup on \mathbf{S} with relations \mathbf{R} . What are the elements of \mathbf{U} ? Write a multiplication table for \mathbf{U} .

Does \mathbf{U} look similar to anything you have seen before?