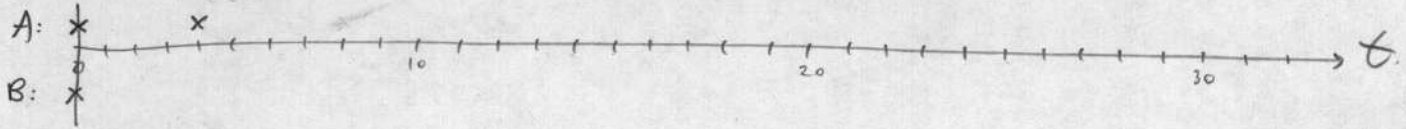


# MATH 5 WORKSHEET : Combined periods & ratios

4/2/07.  
Barnett

Alice & Bob have colds. Alice coughs every 4 seconds while Bob sneezes every 5 seconds. Indicate on this time axis using x symbols the events (they start together):



- How long does it take before A & B are together again? What's the period of the combined 'signal'?
- Now Bob's cold gets better and he sneezes every 6 seconds. What's the combined period?
- What is the general rule?
- Use your rule to get the combined period of the periods 9sec and 15sec?
- What is the period of a signal given by the sum of a sinusoid of period 3.1sec and 3sec?
- What is the period of a signal given by the sum of a sinusoid of period 0.004 sec and one of period 0.005 sec?

Restate this in terms of frequency: signals of freq.  $\text{? Hz}$  and  $\text{? Hz}$  when combined give a signal whose repetition rate is  $\text{? Hz}$

What is the general rule for frequencies?

Apply your rule to get repetition rate of following pairs of tones: 250 Hz and 240 Hz?  
250 Hz and 251 Hz?  
100 Hz and  $100\pi$  Hz?

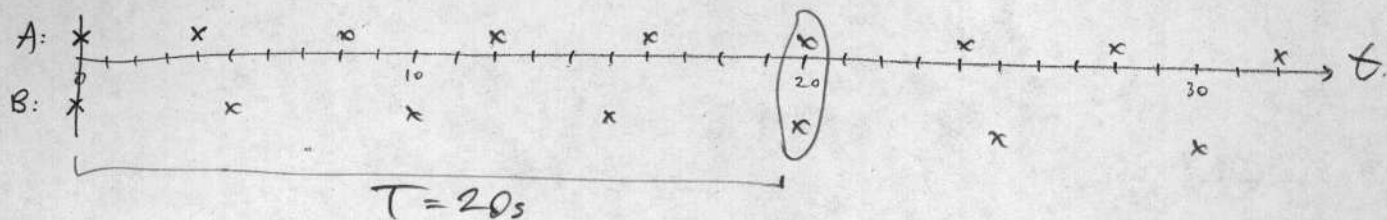
Bonus: what about periods 3.

# MATH 5 WORKSHEET : Combined periods & ratios

## SOLUTIONS

4/2/07.  
Barnett

Alice & Bob have colds. Alice coughs every 4 seconds while Bob sneezes every 5 seconds. Indicate on this time axis using x symbols the events (they start together):



- How long does it take before A & B are together again?  $20\text{ s}$  What's the period of the combined 'signal'?
- Now Bob's cold gets better and he sneezes every 6 seconds. What's the combined period?  $12\text{ s}$  ( $= 3 \times 4$  and  $2 \times 6$ )
- What is the general rule?  $T = \text{lowest common multiple of the periods}$
- Use your rule to get the combined period of the periods 9sec and 15sec?  $45\text{ s}$   
Tricky since not integers! Work in units of  $\frac{1}{10}$  sec and you get  $\text{LCM}(30, 31) = 930$ .  $\leftarrow 3.1\text{ sec and } 3\text{ sec?}$   $93.0\text{ s}$
- What is the period of a signal given by the sum of a sinusoid of period 0.004 sec and one of period 0.005 sec? Work in units of 0.001s  
In these units, periods are 4 & 5, so get 20 as above, so answer is  $\frac{20}{1000}\text{ s} = 0.02\text{ s}$
- Restate this in terms of frequency: signals of freq.  $250\text{ Hz}$  and  $200\text{ Hz}$  when combined give a signal whose repetition rate is  $50\text{ Hz}$   
since  $\frac{1}{0.004} = 250$  etc.  $\frac{1}{0.02} = 50$ .
- What is the general rule for frequencies? 50 is the Greatest common divisor of 250 & 200.

Apply your rule to get repetition rate of following pairs of tones: 250 Hz and 240 Hz?

$$\text{GCD}(250, 240) = 10\text{ Hz}$$

$$\text{GCD}(250, 251) = 1\text{ Hz}$$

250 Hz and 251 Hz?

The ratio  $\frac{100\pi}{100} = \pi$  is an irrational  $\Rightarrow$  the signal never (quite) repeats! 100 Hz and  $100\pi$  Hz?