Math 5: Music and Sound FALL 2010: Final

3 hours, 9 questions, 80 points total

Try to show working. Heed the points available for each question. Do try the bonus once the rest is ok. The last page has useful information. Good luck, have fun, and it was great to have you in the course!

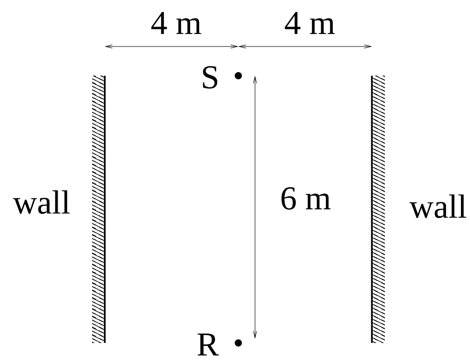
1. [7 points]

   (a) In the space below, draw a spacetime diagram (labeling axes) showing how it is possible for a fixed listener to hear a sound *higher* in frequency than emitted by a moving source.

   (b) What speed does the source need to move, and in what direction, so that the observed pitch is a perfect 4th higher than the source pitch? (You may use *just intonation*, since it's simplest)

2. [9 points] Steve and Rachel stand at opposite ends of a tunnel 6 m in length, as shown below in plan view.

   (a) Compute the travel times of the direct, and of the once-reflected, sound paths from S to R.



   (b) What is the *lowest* pure tone frequency emitted by Steve that would cause destructive interference of these two paths to Rachel?

   (c) Steve claps once. How long after the direct sound arrives does the *second* distinct echo occur? (not the first echo which you computed in a). On the diagram, construct a ray path from S to R corresponding to this second echo.

   (d) [BONUS] Describe (or sketch) the *tail* (long time decaying part) of the echo signal Rachel hears, giving any new perceived pitches resulting from this acoustic environment.

3. [9 points] Two violinists are playing their E strings (with no fingers on the strings), which are supposed to sound the note E5.

    (a) Compute the frequency of the equal-tempered note E5.

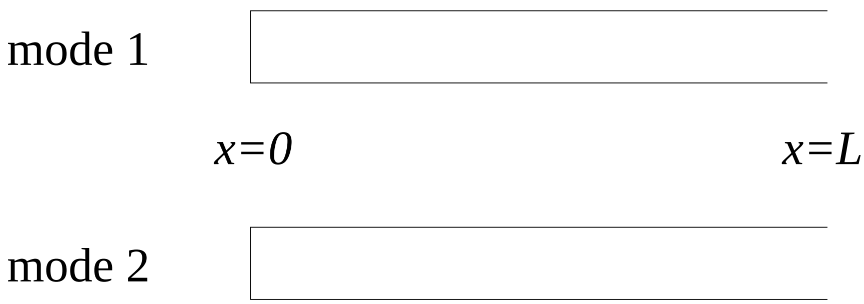
    (b) One of the violinists used equal-tempered tuning while the other tuned using Pythagorean tuning relative to their A4 string. How many cents different are they, and which one is sharp?

    (c) Assuming each player produces something close to a pure tone, describe what you'll hear when they play their notes *together*. Give *all* new frequencies of phenomena perceived (but you don't need to give formulae).

    (d) One of the violinists now wishes to tune (for some reason) their E string *up* an equal-tempered tritone. By what factor do they need to change the tension in the string?

4. [10 points] The adult male human vocal tract can be modeled by a closed-open pipe $L = 0.17$ m long.

(a) Sketch the first two modes showing graphs of pressure amplitude vs position, over the pipes below, labeling nodes and anti-nodes:



(b) Compute the formant frequencies F1 and F2 using this model.

(c) Sketch a spectrum that could be produced by this male when singing a low note with pitch 100 Hz. Label your axes:

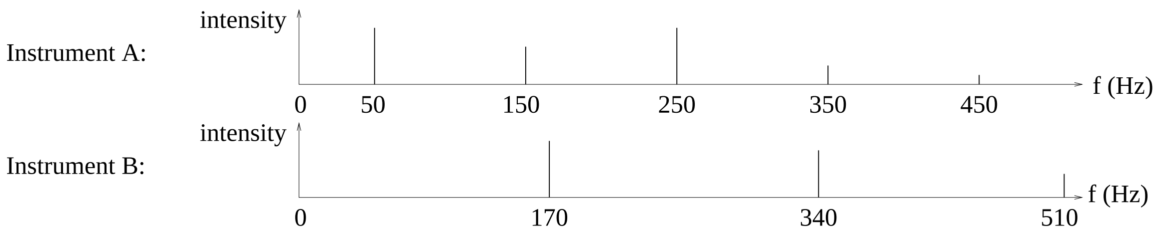
(d) Give a location (e.g. $x = L/2$) where locally *constricting* the vocal tract would cause each of the following formant changes.

- lower both F1 and F2 together:

- lower F1 while raising F2:

5. [9 points] Consider two wind instruments A and B which are based upon pipes of uniform width.

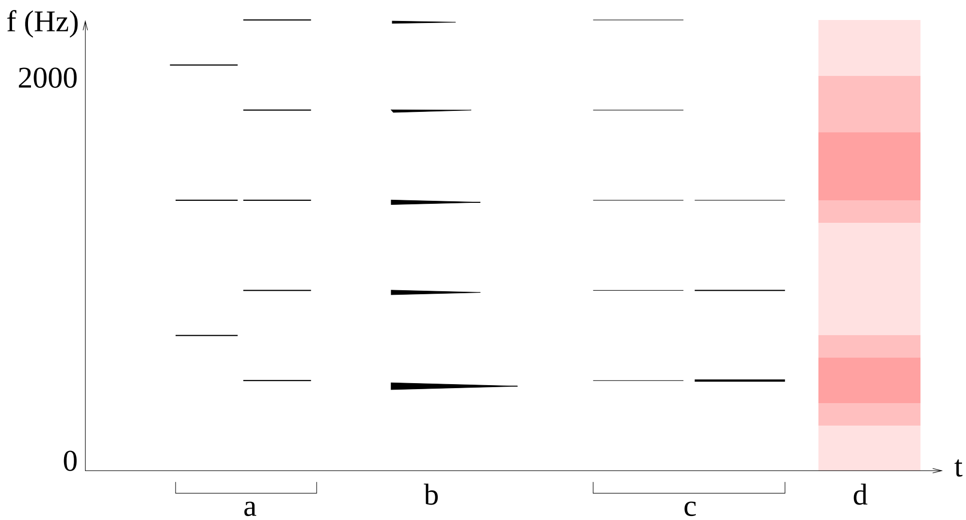
(a) For each instrument, using the spectra below, calculate the pipe length (ignore end corrections) and state whether the end conditions of the pipe are closest to open-open or closed-open.



(b) Using the spectrogram of slapping (impulsive excitation) of instrument A, the lowest 50 Hz mode amplitude is observed to decay 20 dB in 0.1 sec. Compute the Q factor of this mode.

(c) Within what range of pure tone frequencies does the mode discussed in b) get excited with at least half its maximum intensity?

6. [10 points] A spectrogram is shown for a sequence of several sounds. For time periods a, b, c, and d below, describe what might be heard, stating any changes in pitch (up or down?), timbre, and overall amplitude, that occur within that time period:



What instrument or action is most likely to produce sound b? Does it have a perceived pitch?

What instrument or action is most likely to produce sound d? Does it have a perceived pitch?

7. [10 points] A guitar string of fundamental frequency 200 Hz is plucked 1/3 of the way up from the bridge.

   (a) By computing the excitation coefficients $\alpha_1, \alpha_2, \ldots, \alpha_6$, or otherwise, plot a spectrum that could be produced acoustically (go only up to 1200 Hz):

   (b) The player's finger now *lightly* touches the string 3/4 of the way up (i.e. 1/4 of the way along from the neck end), and plucks as before. Plot the acoustic spectrum produced:

   What is the perceived pitch? (leave this as a frequency)

   (c) The player now instead *heavily* presses the string at the same point as in b), pressing it against the fingerboard, and plucks as before. What now is the perceived pitch? (leave this as a frequency)

   (d) Removing the finger as in a), the guitarist switches on an electric pickup 1/2 way along the string, and plucks as before. Plot a spectrum of the electrical signal now sent to her amplifier:

8. [8 points] A tuning fork is struck and produces a pure sinusoid at 300 Hz. A listener is a distance 2 m from the tuning fork.

   (a) Initially the tuning fork radiates 0.005 W acoustic power in all directions. What intensity in dB does the listener hear?

   (b) The Q-factor of the tuning fork is 1000. What is the decay time?

   (c) How long since it was struck with the above initial strength does it take until the intensity at the listener reaches the lower threshold of human hearing which is about 10^{-10} W/m^2 at 300 Hz? (careful, not 10^{-12} W/m^2)

9. [8 points] Short unrelated questions.

(a) A bell produces the following partials all at roughly equal amplitudes: 302, 781, 1168, 1560, 2964 Hz. What 'strike tone' (perceived pitch) frequency is perceived, and why?

(b) Find A such that the pure tone signal $A \sin \omega t + \cos \omega t$ has an amplitude of 2.

(c) When you open a window in a moving car, a Helmholtz resonance may be excited (as when blowing over a bottle). If the window opening is a rectangle 10 cm by 30 cm, the effective neck length is 20 cm, and the resonant frequency is 12.1 Hz, compute the volume of the car cabin in m^3. [BONUS: Comment on its audibility.]

Useful information

$$\omega = 2\pi f$$

$$c = f\lambda$$

$$\text{dB} \; = 10\log_{10}\frac{I}{10^{-12}\text{W/m}^2}$$

$$Q = \pi\frac{\tau}{T}$$

$$\frac{f_{obs}}{f} = \frac{1}{1 - v/c} \quad \text{or} \quad 1 + v/c$$

$$\sin(a+b) = \sin a\cos b + \cos a \sin b$$

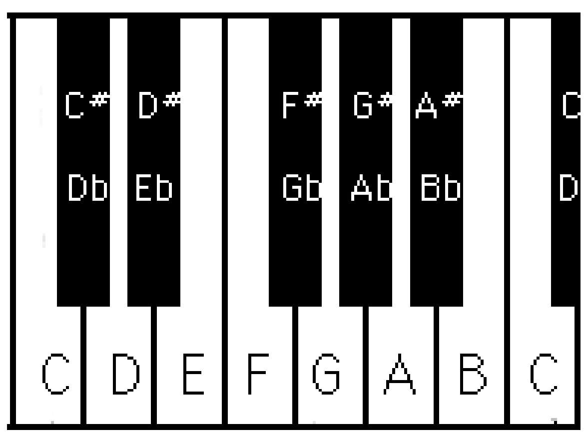
$$\sin a + \sin b = 2\cos(\frac{a-b}{2})\sin(\frac{a+b}{2})$$

$$c_{\text{string}} = \sqrt{\frac{T}{\mu}}$$

$$f_{\text{Helm}} = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}}$$

Intervals by number of semitones:

1. minor second
2. whole tone (major second)
3. minor third
4. major third
5. perfect fourth
6. tritone (augmented fourth)
7. perfect fifth
8. minor sixth
9. major sixth
10. minor seventh
11. major seventh
12. octave



The standard musical pitch A4 is 440 Hz

You can use the speed of sound as 340 m/s.