Math 5: Music and Sound FALL 2008: Final

3 hours, 9 questions, 80 points total

Try to show working. Heed the points available for each question. Try the bonuses once the rest is ok. The last page has useful information. Good luck, have fun, and it was great to have you in the course!

1. [9 points]

   (a) What is the frequency of the pitch C8 (the highest note on the piano) in the equal-tempered system?

   (b) The second column of the touchtone keypad is encoded by frequency 1336 Hz. What musical pitch is this nearest, and what is the error from this pitch in *cents*?

   (c) Construct the frequency ratio between C and the D above it in the Pythagorean tuned C major (diatonic) scale. (Briefly show working)

   (d) If you continued using this Pythagorean construction to compute *all* notes of the chromatic scale, what error in cents would occur when you eventually returned to (and compared against) your starting note?

2. [10 points] An empty 3 liter soda bottle has a neck with radius 1 cm and effective length 3.06 cm.

   (a) Compute the frequency that sounds when someone (*e.g.* Mike Wu) blows across the bottle.

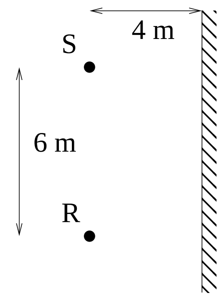
   (b) When the bottle opening is tapped, a pressure signal of the form $Ae^{-t/\tau} \sin 2\pi f t$ is produced. It takes 5 seconds for the *intensity* to drop by 60 dB. Compute the value of the decay time.

   (c) Someone now excites the bottle by sounding a pure tone. What *range* of frequencies would cause at least half the maximum response amplitude inside the bottle? (If you didn't get parts a and/or b, give your answer in symbols.) Sketch a response curve to illustrate this, labeling your axes.

   (d) How much water should be poured into the bottle to change the pitch by one octave? (does it go up or down?)

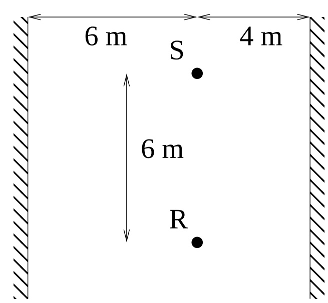
3. [10 points] A speaker cabinet S and listener R are both 4 m from a wall, and 6 m apart, as shown in plan view below.

(a) Sketch the direct and reflected sound paths from S to R and compute their travel times.



(b) What is the lowest pure tone frequency emitted that would cause *destructive interference* of these two paths, for this listener?

(c) A second parallel wall is added 10 m to the left as shown below. Find two *new* travel times which occur. Show any geometric constructions used.



(d) [BONUS] If the speaker emits a loud but short click, what would you expect the *tail* (decaying part) of the echo the listener hears to be like, and why? (diagrams help)

4. [10 points] A child's vocal tract behaves like a closed-open pipe 12 cm long.

   (a) Compute the lowest two formant frequencies assuming the tract is of uniform width.

   (b) Sketch graphs of a spectrum you might hear with this tract shape if the child were...

   singing with pitch 200 Hz: 

   whispering: 

   (be sure to label your axes and give some values on the horizontal axis)

   (c) The child now breathes harmless but dense sulphur hexaflouride which halves the speed of sound in their tract. Redo the new spectrum for the first case above...

   singing with pitch 200 Hz: 

   Explain what has changed and what has not:

   (d) Returning to normal air, the child presses their tongue upwards to constrict the tract 1/3 of the way down from the mouth. What happens to their first two formants? (relative to part a)

(e) Relative to a uniform vocal tract, how could the child easily shape their tract to lower both F1 and F2?

5. [7 points]

(a) A musical note with pitch 200 Hz is accomanied by another, either a perfect fifth above it, or a minor sixth above it. Compare the resulting dissonance for these two situations using Helmholtz's theory (consider only below 1400 Hz)

(b) What is the probable perceived pitch of a sound with partials at 432, 601, 900, 1199, 1435 Hz, and why?

(c) [BONUS] How it can be that the piano sounds better when its octaves are tuned with ratio slightly *greater* than 2:1 ?

6. [9 points] Here you model the sound of the lowest string of the electric bass, which has length 0.8 m.

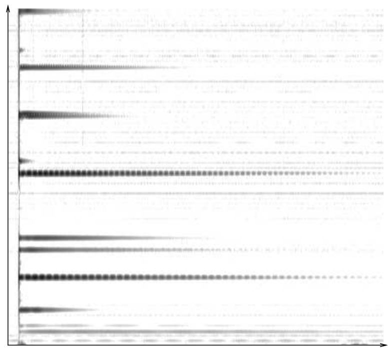
(a) The string produces a note of E1 (41.25 Hz). What is the speed of transverse waves on this string?

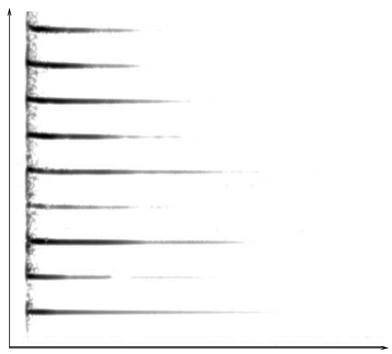
(b) The string is plucked 1/3 of the way up. Compute the excitation amplitudes of the first four modes of the string (are any zero?)

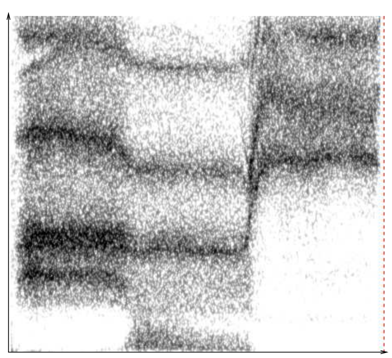
(c) The amplified signal is from an electric pickup placed 1/4 of the way up. Use this and part b) to compute (relative) amplitudes of the first four modes heard, and sketch the resulting spectrum.

(d) A finger is now used to damp (lightly touch) the string 2/5 of the way up. What is the pitch of the sound produced? (note this is true regardless of where it is now plucked)

7. [7 points] The following show actual spectrograms, with frequency (0 to 4000 Hz) vertical, and time (0 to 3 sec) horizontal. Describe pitch (has one?), decay, timbre, and use to state an *instrument* and *method* which could have produced it: (if stuck think about the spectra)

a) 

b) 

c) 

Which of the above, if any, will be periodic signals (at least while they last), and why?

8. [9 points] Short unrelated calculation problems.

(a) What is the *period* of the signal $\sin(100\pi t + \pi/4)$? (as usual, t is time in seconds)

(b) You are a stationary listener. How fast does a source of sound need to travel *towards* you so that its pitch appears to change by an octave?

(c) Compute the intensity in dB of an orchestra radiating 5 W acoustic power in all directions, at a distance of 100 m.

(d) A small animal hears a pure tone of 680 Hz. The phase difference between the signal at its left and right ears is then $\pi/5$ radians. From this find the smallest possible distance between its ears. (Ignore delays due to curvature of the head, *i.e.* assume straight-line travel of sound)

9. [9 points] Explanation questions: points for correct and precise use of concepts. Diagrams can help.

   (a) Explain the difference between frequency and pitch.

   (b) Explain what a Fourier series is and what kind of signals it can and cannot represent.

   (c) Explain how a Tuvan throat singer produces a high-pitched 'whistle-like' melody by singing.

   (d) Explain why the musical interval between a closed-open and open-open pipe of the same length is not exactly an octave (is it bigger or smaller?)

   (e) [BONUS] In class we learned about two ways in which digital (as opposed to analog) sound recording may change (distort) a signal in order to convert it to data. Describe one of these, and the type of distortion produced.

Useful information

$$\omega = 2\pi f$$

$$c = f\lambda$$

$$\text{dB} \; = 10\log_{10}\frac{I}{10^{-12}\text{W/m}^2}$$

$$Q = \pi\frac{\tau}{T}$$

$$\frac{f_{obs}}{f} = \frac{1}{1 - v/c} \quad \text{or} \quad 1 + v/c$$

$$\sin(a+b) = \sin a\cos b + \cos a \sin b$$

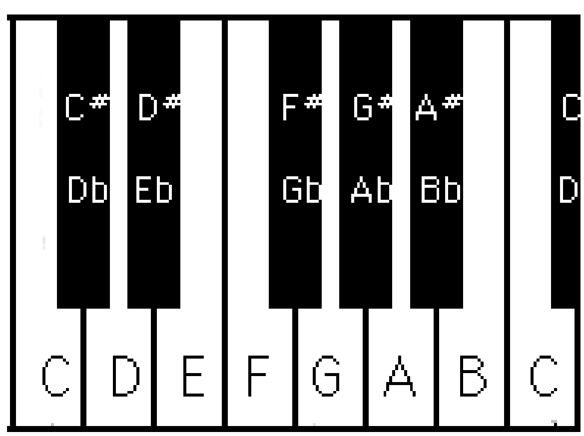
$$\sin a + \sin b = 2\cos(\frac{a-b}{2})\sin(\frac{a+b}{2})$$

$$c_{\text{string}} = \sqrt{\frac{T}{\mu}}$$

$$f_{\text{Helm}} = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}}$$

Intervals by number of semitones:

1. minor second
2. whole tone (major second)
3. minor third
4. major third
5. perfect fourth
6. tritone (augmented fourth)
7. perfect fifth
8. minor sixth
9. major sixth
10. minor seventh
11. major seventh
12. octave



The standard musical pitch A4 is 440 Hz

You can use the speed of sound as 340 m/s.