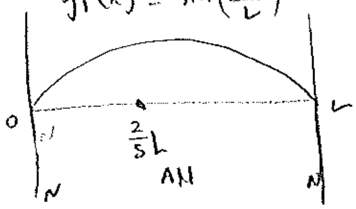
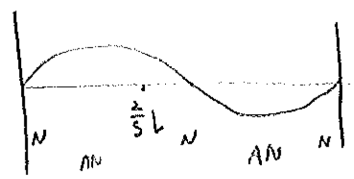
Math 5 (2008)
HW6

1. Modes:

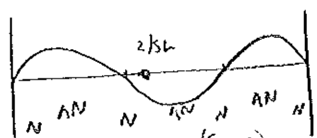
$y_1(x) = \sin\left(\frac{\pi x}{L}\right)$



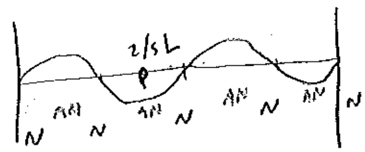
$y_2(x) = \sin\left(\frac{2\pi x}{L}\right)$



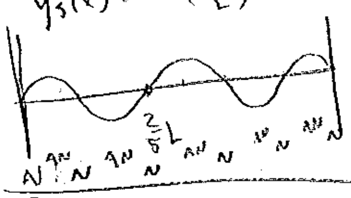
$y_3(x) = \sin\left(\frac{3\pi x}{L}\right)$



$y_4(x) = \sin\left(\frac{4\pi x}{L}\right)$



$y_5(x) = \sin\left(\frac{5\pi x}{L}\right)$



a) Plucked at $x = \frac{2}{5}L$

$\alpha_n = y_n\left(\frac{2}{5}L\right)$

repeating pattern

$\alpha_1 = \sin\left(\frac{2\pi L}{5L}\right) = \sin\left(\frac{2\pi}{5}\right) \approx .951$

$\alpha_2 = \sin\left(\frac{4\pi}{5}\right) \approx .588$

$\alpha_3 = \sin\left(\frac{6\pi}{5}\right) \approx -.588$

$\alpha_4 = \sin\left(\frac{8\pi}{5}\right) \approx -.951$

$\alpha_5 = \sin(2\pi) = 0$

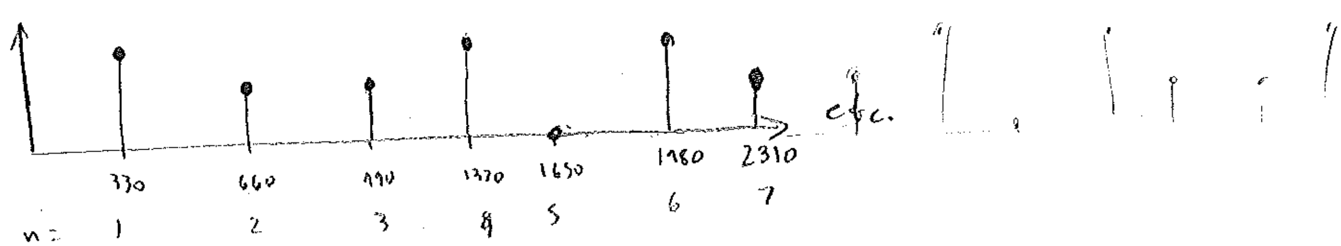
$\alpha_6 = \sin\left(\frac{12\pi}{5}\right) = \sin\left(\frac{2\pi}{5}\right) = \alpha_1 =$

$\alpha_7 = \alpha_2$

etc.

α_n will be 0 for $n = 5k$, where k is an integer greater than 0.

b)



α's are taken from above.

(* This does not account for the $1/n$ factor)

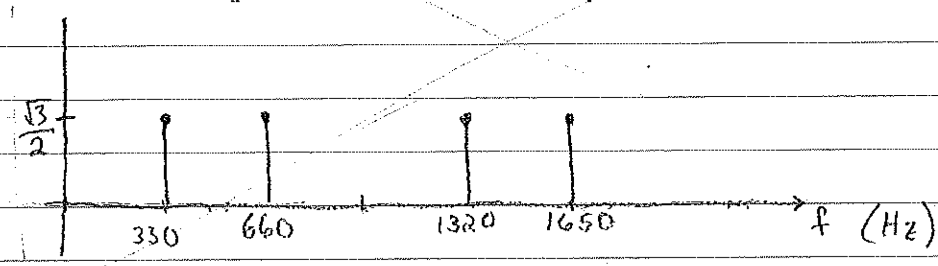
($\boxed{2007}$ only)

b) $f = 330$ Hz

Harmonic content: $f, 2f, 4f, 5f$

330, 660, 1320, 1650

All have magnitude $\pm\frac{\sqrt{3}}{2}$ (just plot absolute values, since amplitude)



c) Finger creates a node at $\frac{1}{4}L$.

All harmonics except multiples of the 4^{th} harmonic will be damped. 4^{th} harmonic already has node there and is unaffected

new fundamental = 1320 Hz

$2f = 2640$

$3f = 3960$

Will perceive the new fundamental 1320 Hz.

↳ $\frac{3}{2}$ above 880 Hz. ⇒ E 6

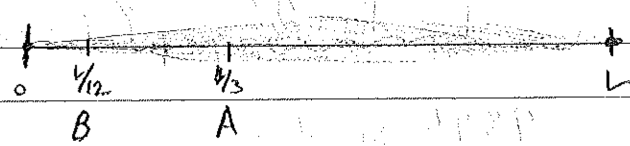
A5

Will hear E 6.

✱ Note: this solution assumes "lightly touching the string" without changing overall length L. Many students assumed this other interpretation, which was also accepted due to the wording of the problem.

(Homework #6 Solutions)

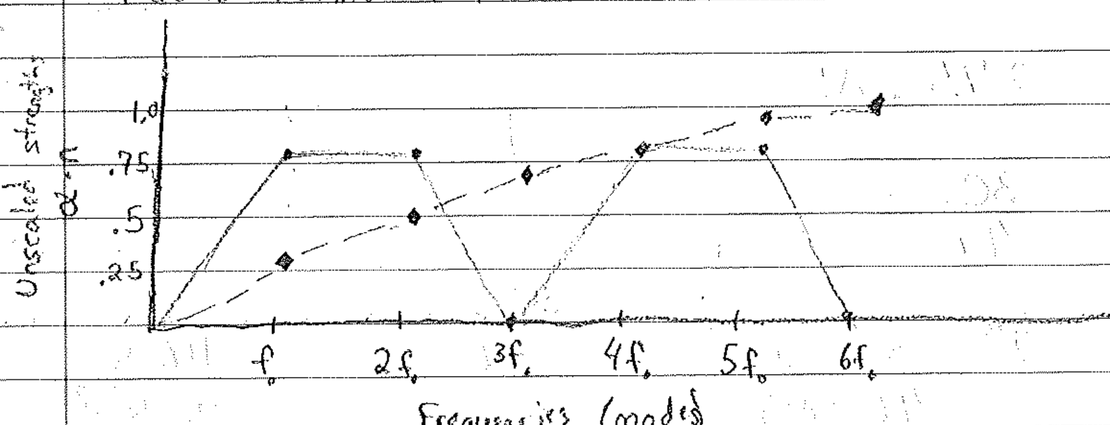
2. a) Electric guitar pickup picks up local oscillations. at position x_p

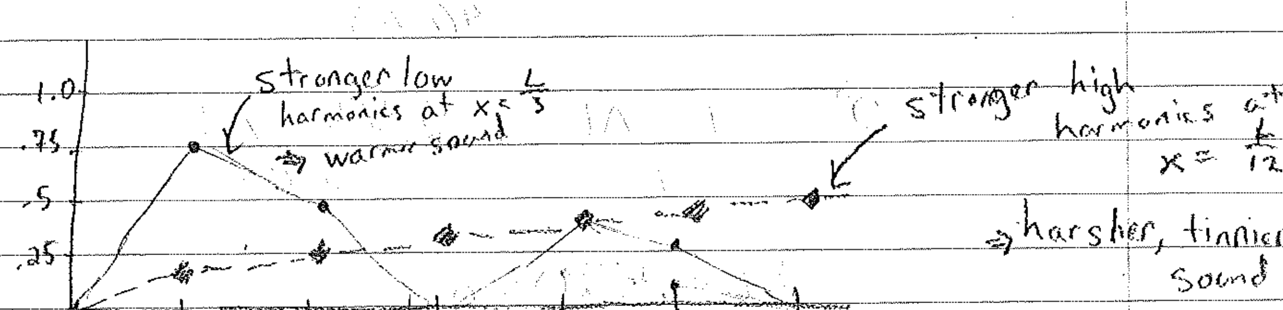


		A	B
Strength =	$\sin\left(\frac{n\pi x}{L}\right)$ —	$\sin\left(\frac{n\pi}{3}\right)$	$\sin\left(\frac{n\pi}{12}\right)$
	α_1	$\frac{\sqrt{3}}{2} = .866$	.259
	α_2	$\frac{\sqrt{3}}{2}$ "	.5
	α_3	0	$.707 = \frac{\sqrt{2}}{2}$
	α_4	$\frac{\sqrt{3}}{2}$	$.866 = \frac{\sqrt{3}}{2}$
	α_5	$\frac{\sqrt{3}}{2}$	.966
	α_6	0	1

- Will pick up same frequencies (harmonics) but have different strengths.
- Plucked harmonics roll of $\alpha \frac{1}{n}$

b)





c) No Signal At pickup A, as freqs excited will be multiples of $n=3$, all of

3. Rule of 18: $f_1 = f_0 \cdot \frac{18}{17}$

Equal-tempered semitone $f_1 = f_0 \cdot 2^{1/12}$ (1 semitone)

$12 \times \frac{\ln\left(\frac{18}{17}\right)}{\ln(2)} = .9895$ semitones

Error in semitones $= .0105 \Rightarrow 1.05$ cents

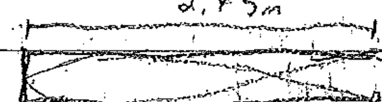
- Assume string tension is unaffected by pressing finger to change length, otherwise fret position wouldn't be directly correlated to pitch. The inclusion of the change in tension would make things messier.

only 2007 | 2007

3. $L = 2.75$ m 1^{st}

2^{nd} mode $Bb2 \Rightarrow 116.5$ Hz
$n=3$

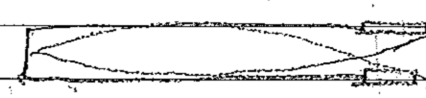
2.75 m



closed tube $f_n = \frac{nC}{4L}$

n = odd

a) $L = 2.75 + \Delta L$



$116.5 = \frac{3C_{sound}}{4L}$

Minor third: $116.5 \cdot 2^{3/12} = \frac{3C_{sound}}{4(L+\Delta L)}$

≈ 3 semitones

$$\frac{\frac{3C_{sound}}{4L}}{\frac{3C_{sound}}{4(L+\Delta L)}} = \frac{116.5}{116.5 \cdot 2^{3/12}}$$

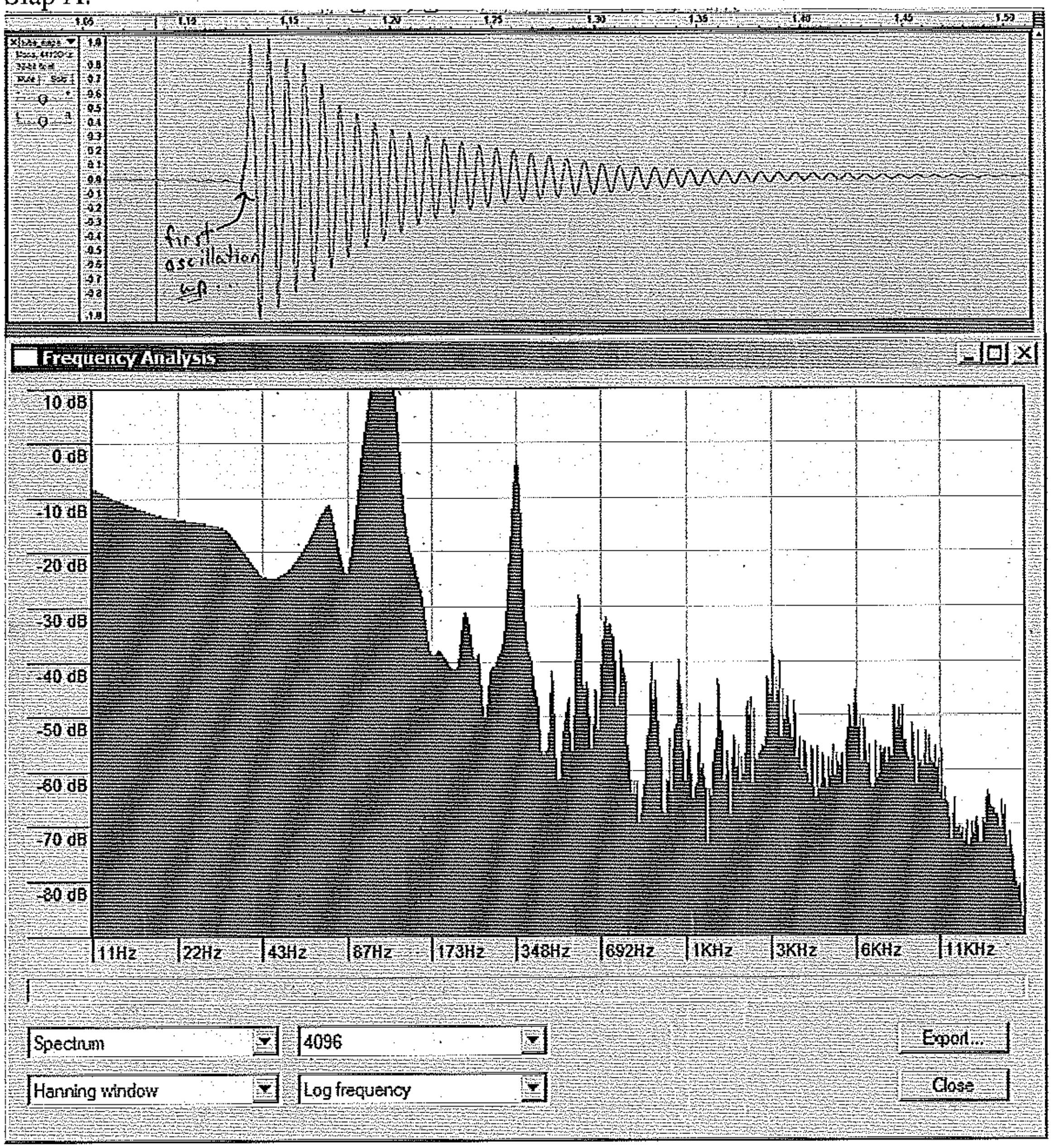
$$2^{3/12} = \frac{L+\Delta L}{L} = \frac{2.75+\Delta L}{2.75}$$

$$\Delta L = .5203 \text{ m}$$

Change in slide position $= \Delta x = \frac{\Delta L}{2}$

4. Analyzed spectrums of Slap A and Slap B in ~~Audioscan~~. Audacity

Slap A:

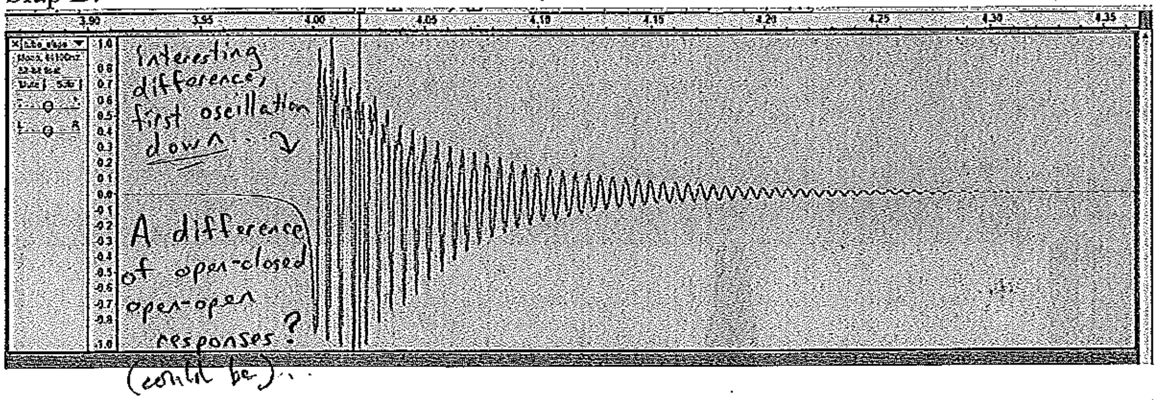


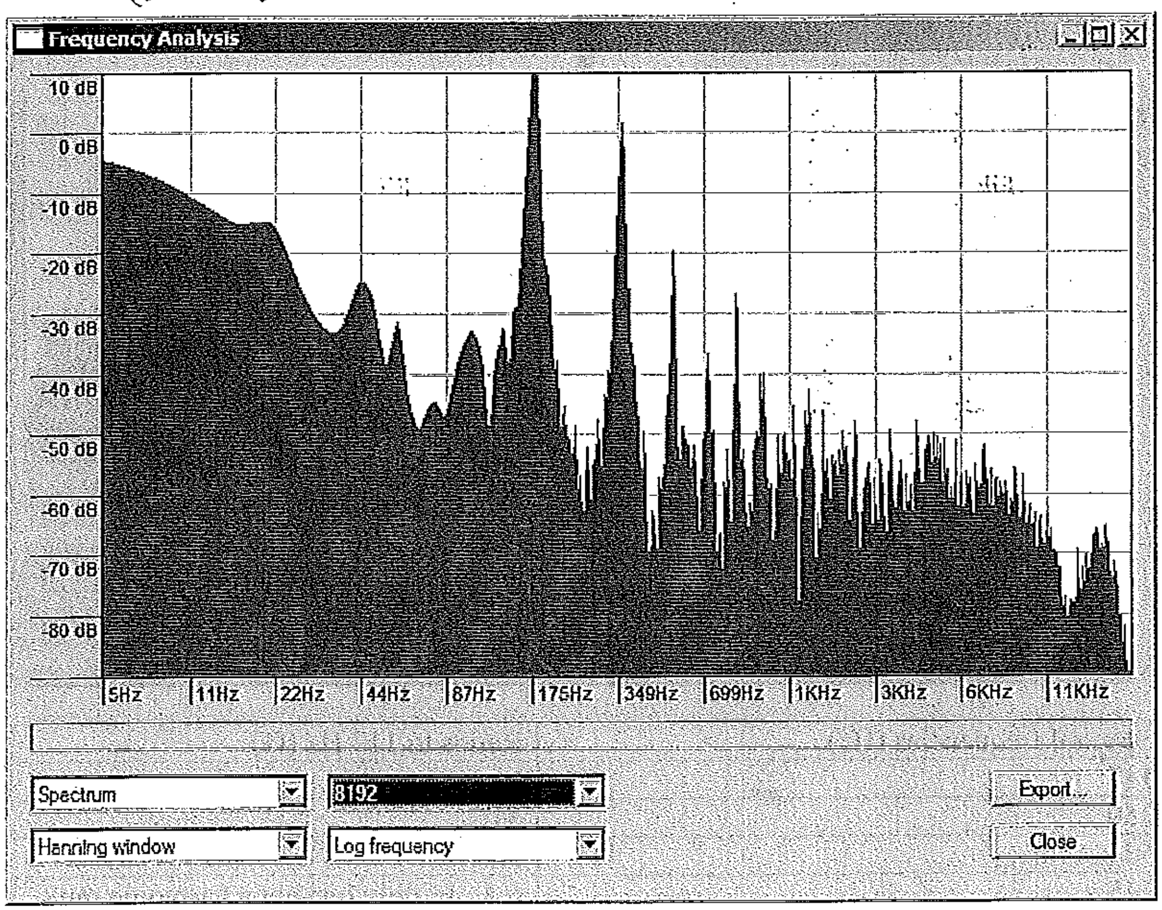
Spectral frequencies (Hz):	Ratios to 117 Hz (f1)
117	1
350	3.0
584	5.0
745	6.4 ← only one not a harmonic mult.
816	7.0
1049	9.0
1306	11.2

Mode frequencies appear to be only odd multiples. This corresponds to the formula for a close tube: f = n * c / (4 * L) where n is any odd integer (1, 3, 5, etc.). This is a one end open, one end closed tube. If the fundamental = 117 = c / (4*L) = 340/ (4*L), then-

L = .726 m, one end open.

Slap B:





Spectral frequencies (Hz):	Ratios to 179 Hz (f1)
179	1
362	2.0
545	3.0
718	4.0
911	5.1
1133 or 1102	6.2 (for 1102 Hz)
1277 or 1344	7.1 (for 1277 Hz)
1444	8.1

Frequencies follow integer multiples. Corresponds to a both ends open tube: $f = n*c/(2*L)$. $f = 179 = c/(2*L)$. So

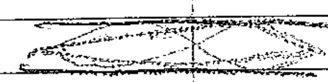
$L = 0.95$ m, both ends open.

5. C3 → 3 semitones up from A2

$f = 110 \times 2^{3/12} = 130.8\ Hz$

a)

Bass flutes are two-sided open pipes.

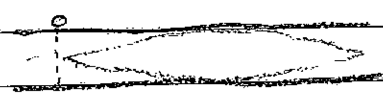


$$f_1 = \frac{c}{2L} \qquad f_n = \frac{nc}{2L}$$

$$130.8 = \frac{340}{2L}$$

$$L = \frac{340}{2 \times 130.8} = 1.30\ m$$

b) First hole placement to go up by whole tone



$$\underset{\frac{c}{2L}}{\underset{\uparrow}{f_2}} = \underset{\frac{c}{2(L-\Delta L)}}{\underset{\uparrow}{f_1}} \cdot 2^{2/12}$$

$$\frac{\cancel{c}/\cancel{2}L}{\cancel{c}/\cancel{2}(L-\Delta L)} = 2^{2/12}$$

$$\frac{1.30}{1.30 - \Delta L} = 2^{2/12}$$

$\Delta L = .159\ m$ from end

c) closed - open pipe $f_1 = \frac{c_s}{4L}$. $L = .49\,m$

$$f_1 = \frac{340\ m/s}{4(.49\,m)} = 173.5\ Hz \approx F3$$

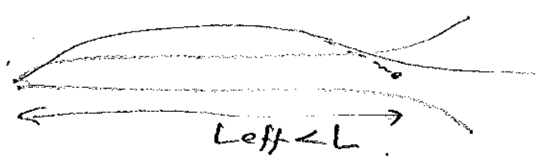
↖ whole step lower!

$$f_{G3} = 490 \cdot 2^{-14/12} = 195.99\ Hz$$

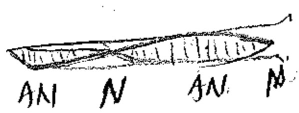
$$L_{calc} = \frac{c}{4f} = \frac{340}{4 \cdot 195.99} = .434\,m$$

↑ functionally the node is here

This makes sense, as the bell shape has a larger width, and that pressure in the pipe would "decay" as the width becomes larger.

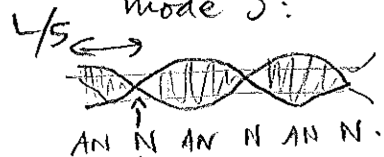


d) mode 2



$f_2 = 3f_1$ but.

mode 3:



$f_3 = 5f_1$

↳ you would want to open at this node, which would allow pipe mode 3 to be excited fully, and would dampen pipe modes 1, 2

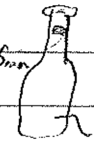
This is at $\frac{1}{5}$ L.

2nd Pipe mode Fundamental : $3f_1$ = octave + fifth above G3 = D5.

3rd : $5f_1$ = 2 octaves + third above G3 = B5.

[2010 Cauc]

[2010]

6. $d = 19.5\,mm$  $5\,cm = l = .05\,m$

$$a = \pi \left(\frac{d}{2}\right)^2 = \pi \cdot \left(\frac{.0195}{2}\right)^2 = 2.99\times10^{-4}\,m^2$$

$$V = 500\,cm^3 = 500\cdot(.01)^3 = 5\times10^{-4}\,m^3$$

a)

Bottle resonating is a Helmholtz resonator.

$$f_{helm} = \frac{c}{2\pi}\sqrt{\frac{a}{lV}} = \frac{340}{2\pi}\sqrt{\frac{2.99\times10^{-4}}{.05\cdot5\times10^{-4}}} = 187\,Hz$$

b) $f_{helm} \propto \sqrt{\frac{1}{V}}$

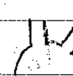
$\frac{3}{2}f \longrightarrow \frac{4}{9}V$

Frequency increases — Volume decreases

Water decreases resonant air volume.

water added $= V - \frac{4}{9}V = \frac{5}{9}V$

$= 278\,cm^3$

c)  2 necks doubles a so f_{helm} increases by factor $\sqrt{2}$ to $264.7\,Hz$, tritone up. equal-tempered 6 semis

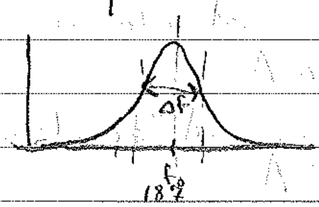
(corrected so Δf = full width at half-max response).

2007, 2008 only.

c) ~~Δf~~ $\rightarrow$

$\Delta f = \frac{1}{2}$ semitone, ie ± 25 cents from center freq.

$\frac{1}{4}$ semitone



$$\Delta f = f\cdot 2^{\frac{1}{4}\cdot\frac{1}{12}} - f\cdot 2^{-\frac{1}{4}\cdot\frac{1}{12}}$$

$$\approx (2^{\frac{1}{2}\cdot\frac{1}{12}} - 1)\,f$$

$$= 0.029\,f \approx 5.48\,Hz$$

$$Q = \frac{f_0}{\Delta f} = \frac{187}{5.5} = 34.1$$

$$Q = 34.1 = \frac{\pi\cdot\tau}{T} = \pi\cdot\tau\cdot f_0 = \pi\cdot\tau\cdot187$$

$$\tau = \frac{34.1}{\pi\cdot187} = .058\ (s)$$

7. a) $f_{o\,violin} = \frac{c_{string}}{2L}$ $f_{o\,flute} = \frac{c_{air}}{2L}$

$c_{string} < c_{air}$ (although this not need always be the case if make tension high enough).

So if L is comparable, violin will play lower due to the slower speed of sound in the string than in the air. (The effective length of the flute may also be smaller than the physical length, due to opening finger holes).

b) $f_{o\,bottle} = \frac{c}{2\pi}\sqrt{\frac{a}{\ell v}}$ $f_{flute} = \frac{c}{2L} \propto \frac{1}{L}$

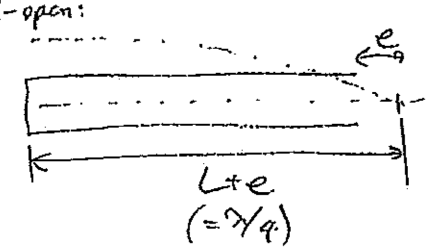
$f_{bottle} \propto \frac{1}{\sqrt{L}}$ $v \simeq \underbrace{L}_{\text{length}} \cdot \underbrace{A}_{\text{cross-sectional area}}$

Bottle is a Helmholtz resonator, has a different formula for its mode frequencies than the flute, and can produce <u>much</u> lower frequencies than an equal length open-open (or open-closed, even) tube.

2008/2009 only [c) A lot of our instrument & timbral perception must come from the attack-sustain-decay-release properties of the instrument, and not solely on the harmonic content.]

[Math 5 HW ~~#~~6. Solutions.] ~~11/12/08~~ 2010.

8. a) closed-open:

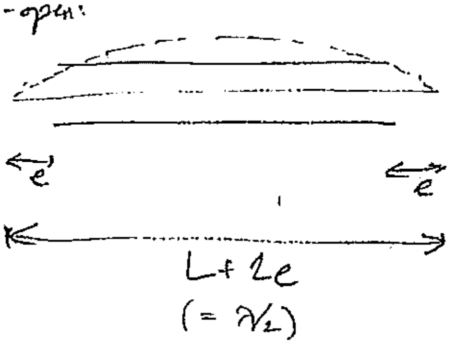


$$166.7 = f = \frac{c}{4L_{eff}} = \frac{c}{4(L+e)}$$

give them symbols, ie easier to do algebra.

$$326.9 = F = \frac{c}{2L_{eff}} = \frac{c}{2(L+2e)}$$

open-open:



rearrange the 2 eqns:

$$4L + 4e = \frac{c}{f}$$

$$2L + 4e = \frac{c}{F}$$

mult by 2.

$$4L + 8e = \frac{2c}{F}$$

subtract: $2L = \frac{c}{F} - \frac{c}{f}$

ie $L = \frac{c}{2}\left(\frac{1}{F} - \frac{1}{f}\right)$ insert data f, F

≈ 0.4998 m ≈ 50 cm.

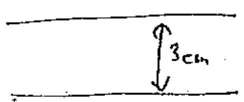
subtract top eqn:

$$4e = c\left(\frac{2}{F} - \frac{1}{f}\right)$$

$$e = \frac{c}{4}\left(\frac{2}{F} - \frac{1}{f}\right) \approx 0.0101 \text{ m} \approx 1 \text{ cm}$$

(end correction, so diam ~ 2.5cm?)

b)



$e = (0.4)3\text{cm} \approx 1.2\text{cm}$

$L_{eff} = L + 2e$ since open-open

C4 is ~~8~~ 9 semitones below A4 = 440Hz.

so $f = 440\left(2^{-9/12}\right) \doteq \cancel{130.8}$ 261.6 Hz. $= \frac{c}{2L_{eff}}$ (o-o pipe)

so $L \doteq L_{eff} - 2e = \frac{c}{2f} - 2e \approx \cancel{1.300}\ 0.650 - 0.024$ m

≈ 0.626 m 62.6 cm. ~~≈ 1.276 m~~