Math 5 HW3 Solutions

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1. [8pts]

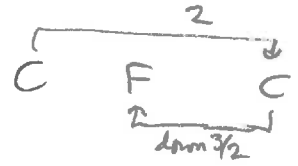
a) 3 semi's goes 4 times into octave (12 semi's) : $(6/5)^4$ is interval of 4 just minor 3rds.

Compute # cents for it : $\text{cents} = 1200\,\frac{\log (6/5)^4}{\log 2} = 1262.6\ldots$

(1200 is true octave)

$\Rightarrow$ 62.6 cents sharp.

b) F is perfect 5th (3:2) down from C an octave up:

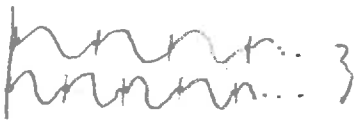


So $f_F = 262\,(2)\cdot 2/3 = 349.33\ldots$ Hz

Equal-tempered is 5 semis above C: $f_F = 262 \cdot 2^{5/12} = 349.73\ldots$ Hz

v. close!

c) 294 Hz & 490 Hz are far apart (> 15Hz) $\Rightarrow$ not a beating effect question.



when do signals first hit start of period together again?

start together

[See HW1 qn. 8] ← here you discovered the rule

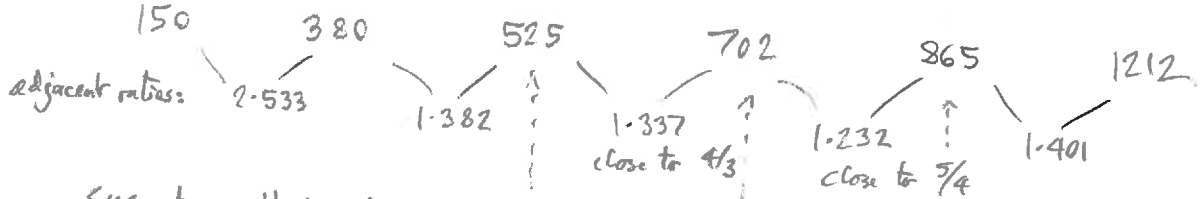
combined repetition period = LCM (T_1, T_2) ← the two periods

or " " freq = GCD (f_1, f_2)

So GCD (294, 490) = 98 Hz (lines up every 3 reps. of one, 5 reps. of other)

GCD (294, 492) = 6 Hz (takes much longer since not very related!).

2. [6pts]



suggests multiple of f: 3 : 4 : 5 & others not in any harmonic series

check $\frac{525}{3} = 175.0$

$\frac{702}{4} = 175.5$

$\frac{865}{5} = 173$

} all within 1% of 174.

so missing fundamental (perceived pitch) is $\approx$ 174–175Hz

$\approx$ 16.0 semis below A4

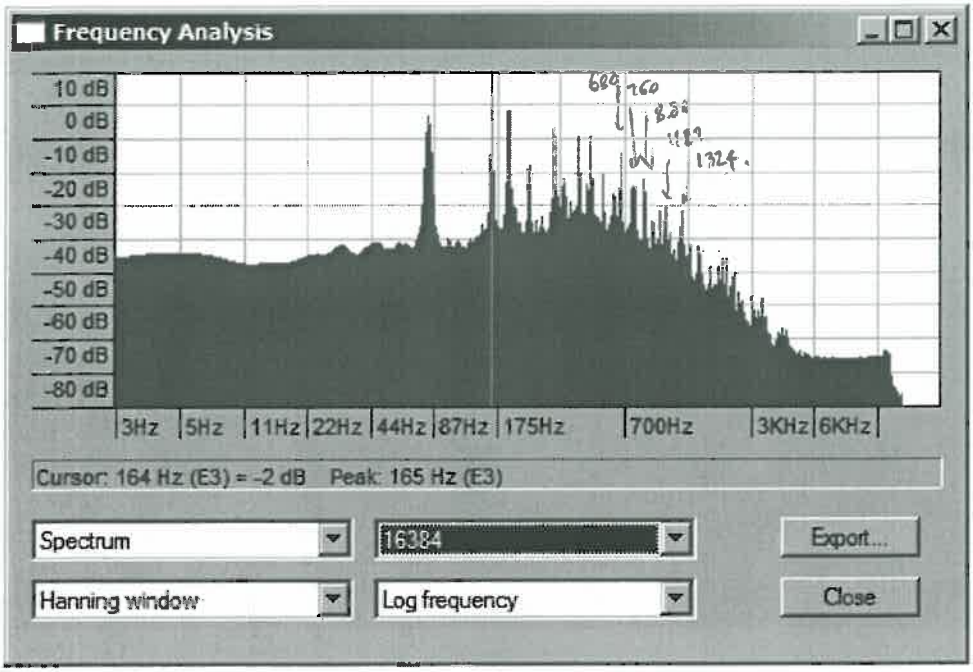
$\approx$ F3

New partial at 963 Hz is roughly 5.53 times the perceived 174 so is not close to an integer multiple, will weaken the perceived pitch impression.

3. [6 points] Audacity

Plot spectrum on ~~[illegible]~~, log scale, 16384 steps:

[Stephen Seaules].



Main peak frequencies present (Hz)	~ Integer multiples of 220 Hz
82	-
165	-
198	-
248	-
330	-
427	**1.94** ← (may not be close enough)
488	2.21
545	2.47
680	**3.09**
760	3.45
880	**4.00**
1112	**5.05**
1324	**6.02**

harmonics of 220 Hz. ↓

3

4

5

6

Contains very close to integer multiples of 220 Hz for 4th, 5th, and 6th harmonics as well as fairly close integer multiples for the 2nd and 3rd harmonics above 220 Hz. The other prominent peak frequencies are scattered around in a random not-harmonically related way. Our ear picks up on frequency multiples and computes the missing fundamental least common denominator for the frequencies present. Though there may be other ratios present, 220 Hz has at least 5 or 6 of its harmonics (could be more up the spectrum) represented in the frequency spectrum, so this is what our ear picks up on.

[MS (HW3]

4. [6pts]

300 600 900 1200 1500 1800 ← Harmonics of lower note.

a) $\frac{8}{5} \cdot 300 \rightarrow 480$; 960 (1 diss.) ; 1440 (1 diss) ; 1920 (1 diss)

(3 fully dissonant) (no partially diss)

b) 317 (1 diss) ; 636 (1 diss) ; 954 (1 diss) ; 1271 (1 diss) ; 1589 (1 diss) ; 1907 (1 diss)

All partials dissonant! (5 fully diss).

So min 6th more harmonics

5. [6pts]

a) $\lambda = \frac{c}{f} = 17\text{m}$ for $f = 20\text{Hz}$, $0.0227\text{m} = 2.27\text{cm}$ for 15kHz.

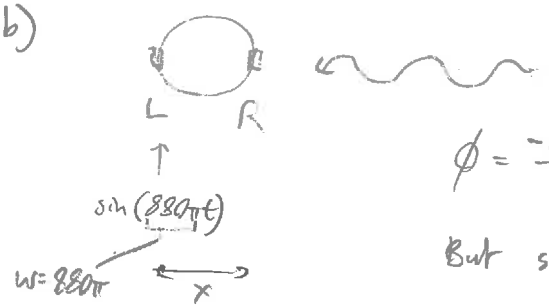
b) $f = c/\lambda = \frac{340}{0.304} = 1118\text{ Hz}$

c) $f = c/\lambda = \frac{1500}{0.001} = 1500000\text{ Hz}$! ($1.5\times10^6\text{ Hz} = 1.5\text{ MHz}$, more like radio)

6. [6pts]

a) $t = \frac{\text{dist}}{\text{speed}} = \frac{0.2}{340} = 0.000588\text{ sec} = 0.588\text{ ms}$

b)



R ear is ahead since signal reaches it first, so we expect $\phi > 0$.

$$\phi = \frac{-\omega x}{c} = -\frac{880\pi(0.2)}{340} = -1.63\text{ rad.}$$

But since R is ahead we set sign to positive: $+1.63$ rad

c) Out of phase means $\phi = \pi$ so set $\frac{\omega x}{c} = \pi$ ie $\omega = \frac{\pi c}{x} = \frac{\pi 340}{0.2}$

ie $f = \frac{\omega}{2\pi} = \frac{340}{2(0.2)} = 850\text{Hz}$

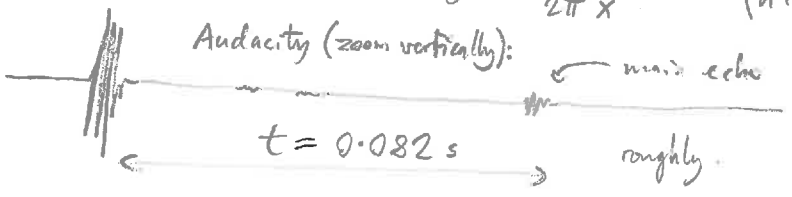
note: $\frac{\text{wavelength}}{2}$ = head size.

[Note there will be $\phi = 3\pi, 5\pi, \ldots$ also out of phase but these must occur at higher f's.]

Bonus: $\phi = (2n+1)\pi$ so $f = \frac{(2n+1)\pi c}{2\pi x} = (n+\tfrac{1}{2})\frac{c}{x} = 850 + 1700n$ Hz (n integer)

7. [8pts]

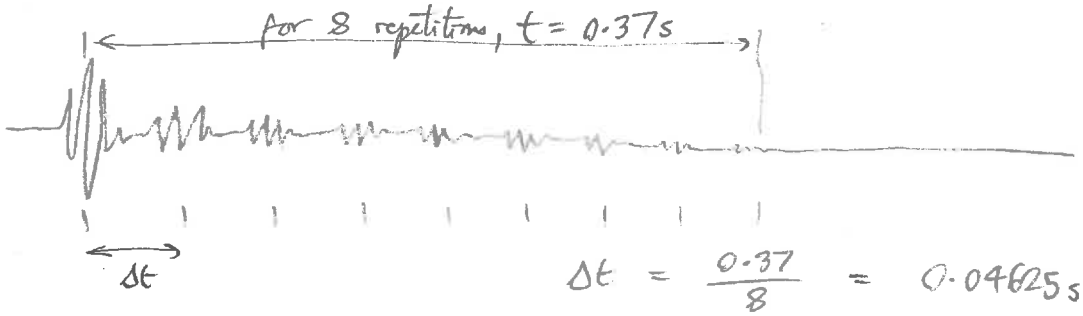
a)



round-trip time = $t = \frac{2L}{c}$

$\Rightarrow L = \frac{ct}{2} = 13.9\text{ m}$

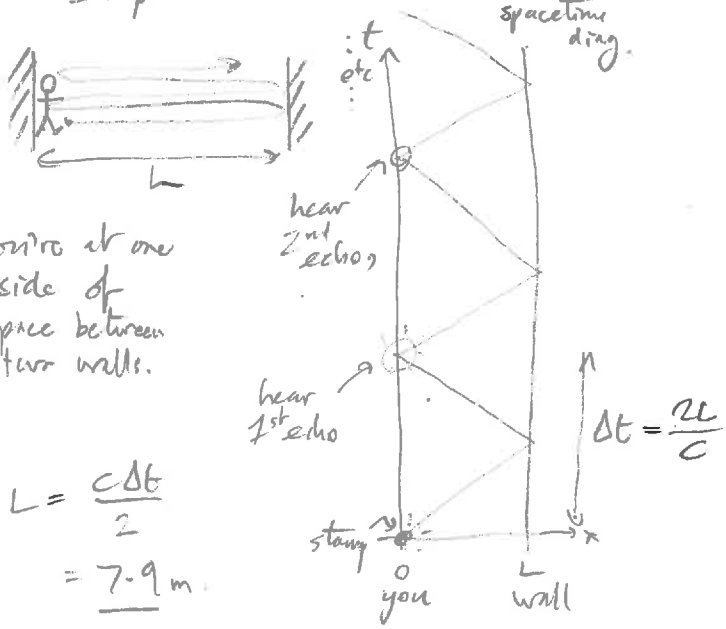
[7] b)



$$\Delta t = \frac{0.37}{8} = 0.04625\,s$$

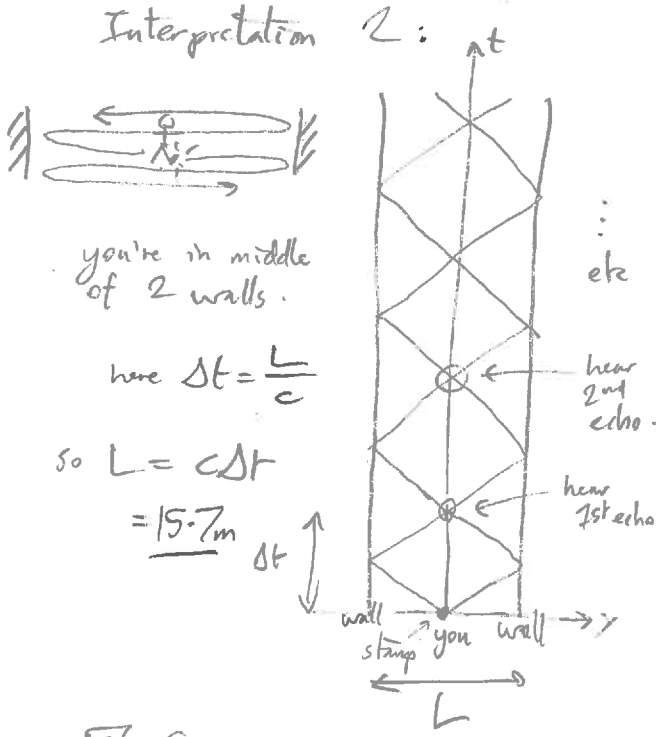
Can't get multiple reflections from one wall ⇒ suggest two walls, parallel.

Interpretation 1:



$$L = \frac{c\Delta t}{2} = 7.9\,m$$

Interpretation 2:

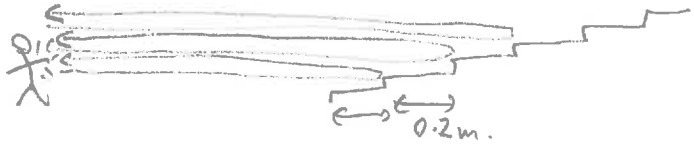


here $\Delta t = \frac{L}{c}$

so $L = c\Delta t = 15.7\,m$

Interpretation 3: multiple walls as in qu. [8] (although unlikely).

[8] [5pts]

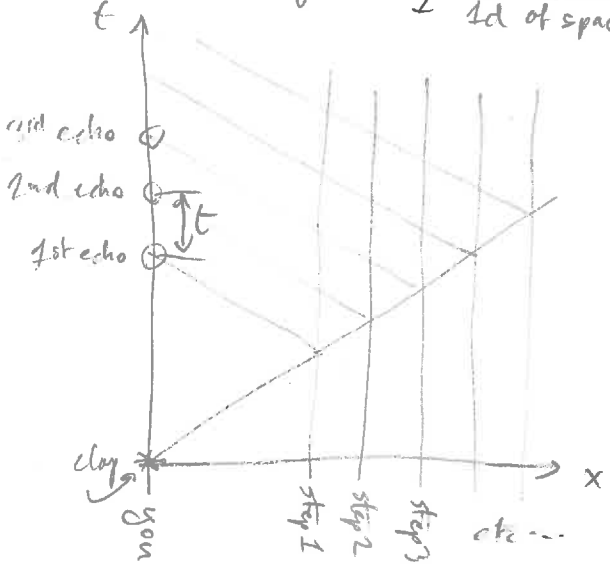


extra travel dist = 0.4m
(between echoes from successive steps)

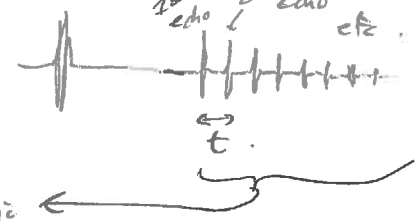
$$t = \frac{0.4}{340} = 0.001176\,s$$

Spacetime diag:

note staircase (2d object) cannot be drawn on 1d of space here:



So, signal will be



approximately periodic
w/ period t

⇒ pitch $f = \frac{1}{t} = 850\,Hz$

(40 cent sharp of Ab5)