Math 5: Music and Sound. Homework 3

due Wed Oct 12 . . . but best if do relevant questions after each lecture

Below you may take the speed of sound in air to be $c = 340$ m/s.

1. Review tuning systems.
   (a) The 'just' minor third is 6:5. However, repeatedly going up by this interval a certain number of times (how many?) is supposed to give an octave, but doesn't quite. How far off in cents is the not-quite-octave you got using the just minor thirds from the true octave?
   (b) Starting from a key note of C = 262 Hz, compute the frequency of the Pythagorean-tuned F in the C-major scale.
   (c) What is the period of the signal produced by adding pure tones at 294 and 490 Hz? What is it if now the second is changed almost imperceptably to 492 Hz? (The fact that these two pairs would sound very similar, i.e. like a major sixth, demolishes the idea that the ear detects strict periodicity.)
2. The spectrum of a bell[1] contains the following main partials (in Hz): 150, 380, 525, 702, 865, 1212 Predict with reasons what apparent frequency ('strike tone'), and corresponding musical pitch, you would hear. [Hint: one way is to take ratios between all pairs of partials]. Would an additional partial at 963 Hz strengthen or weaken this impression and why?
3. When I listen to the bell sound in AD/DC's "Hells Bells" (the first Aural Posting, also on HW page) I hear a strong impression of A3 (220 Hz), and I think you will too[2]. However there is *no partial* at 220 Hz or anywhere very near it. Please explain this (you will want to our sound analysis tools).
4. Compute how the Helmholtz theory of dissonance would rate the consonance or dissonance of the following intervals, counting only the partials up to 2000 Hz. Assume the lower note is at 300 Hz in each case, as in the worksheet. Here's a scoring system that's a bit more realistic than the one in class: count 1 for a fully dissonant pair of partials (less than 10% different), and 0.5 for a marginally dissonant pair (between 10% and 15% different). Let's say partials closer than 15 Hz (the max beat frequency) don't count as dissonant at all. Please state how many fully and slightly dissonant partials there are in each case.
   (a) the minor 6th (use just tuning with 5:8)
   (b) the minor 2nd, *i.e.* one semitone (use equal-tempered tuning)

   Which interval is more harmonious according to the theory?
5. Wavelengths and frequencies.
   (a) What is the range of wavelengths in air for a human hearing range of 20 Hz to 15,000 Hz ?
   (b) Find the frequency of a pure tone that has a wavelength of one foot (0.304 m)
   (c) What is the frequency of ultrasound used for medical imaging, wavelength 1 mm ? (you'll need that the wave speed in human tissue is around 1500 m/s).

[1] I took this example by tweaking the partials of the Pummerin 1951 bell you can hear at `http://www.hibberts.co.uk/belllist.htm`

[2] There's also a low E3 but I find it weaker

6. Traveling waves. Assume the human ears are about 0.2 m apart, and that the head poses no obstruction to the propagation of sound. Let's imagine sound waves arrive from your right (*i.e.* 90 degrees to the right of straight ahead).

   (a) Compute the time delay between a signal arriving at the left and right ears.

   (b) If the signal arriving at the left is $\sin(880\pi t)$, compute the *phase* at the right ear, *i.e.* the value ϕ in in right ear signal $\sin(880\pi t + \phi)$. [Be careful to check if the phase is ahead or behind]

   (c) What is the *lowest* pure tone frequency such that the signals at the two ears are exactly out of phase (π phase difference) ? [BONUS: what is the set of *all* such frequencies?]

7. Here are two echoes I recorded on or near campus. Analyse them with `audacity` in order to answer the questions. [Hint: you will want to zoom in vertically by clicking on the y-axis until you can measure accurately what you want]

   (a) 'Mystery echo A': I was standing in front of a wall which gave the strongest reflection (ignore the other weaker ones). How far away was I?

   (b) 'Mystery echo B': I stamped my foot a few times (the last one was best). After each stamp you can hear a 'flutter echo', *i.e. multiple* repetitions of the echo. What arrangement of walls might be causing the echo? (Ask if stuck) Then please compute relevant distance(s).

8. If you stand in front of a long stone staircase with steps of width 0.2 m, some distance away, and clap, the echo consists of a sequence of impulses each reflected from a vertical face of a single stone step. Assume all the impulses travel horizontally (ignore vertical motion). Draw a diagram of the echo paths returning to you (hint: what extra round-trip travel distance is added when you go up one step?) , and sketch a graph of a plausible signal you would hear. Compute the frequency you will perceive in the reflected echo. (there are theories that Mayan pyramids encoded bird sounds this way!)