Homework #2 Solutions (2008).

1. 5 points

Hear beats when 1 semitone $\leq$ 15 Hz

Highest note which hear beats w/ one semitone above:

$$f_1 \times 2^{(1/12)} = f_1 + 15$$

$$f_1 \left[2^{(1/12)} - 1 \right] = 15$$

$$f_1 = \frac{15}{2^{(1/12)} - 1} = 252.26 \text{ Hz.}$$

Check:

$252.3 \times 2^{1/12} = 267.3$
$252.3 + 15 = 267.3$ ✓

C4 = 261.6 Hz
B3 = 246.9 Hz

$\longrightarrow$ 37 cents sharp of these notes

B3 = 246.9 Hz is highest note which produces audible beats with the semitone above it.

2. a.) For even functions:

7 points 2 points

$$f(t) = f(-t)$$

Note: g(t) doesn't need to have any symmetry.

substitute $-t$ for t.

$$e(t) = \frac{1}{2}\left(g(t) + g(-t)\right)$$

$$e(-t) = \frac{1}{2}\left(g(-t) + g(t)\right) = e(t) \quad \checkmark$$

$\therefore$ $e(t)$ is even

$\therefore$ "Therefore,"

b.) For odd functions:

2 points

$$f(t) = -f(-t)$$

again, subst. $-t$

$$o(t) = \frac{1}{2}\left(g(t) - g(-t)\right)$$

$$o(-t) = \frac{1}{2}\left(g(-t) - g(t)\right) = -\frac{1}{2}\left(g(t) - g(-t)\right) = -o(t) \checkmark$$

$\therefore$ $o(t)$ is odd

c.) $e(t) + o(t) = ?$

1 point

$$\frac{1}{2}\left(g(t) + g(-t)\right) + \frac{1}{2}\left(g(t) - g(-t)\right) =$$

$$= \frac{1}{2}g(t) + \frac{1}{2}g(t) + \cancel{\frac{1}{2}g(-t)} - \cancel{\frac{1}{2}g(-t)} =$$

$$= g(t)$$

$$\therefore \; e(t) + o(t) = g(t)$$

d) $g(t) = (1-t)^2$

2 points

$$e(t) = \frac{1}{2}\left(g(t) + g(-t)\right)$$
$$= \frac{1}{2}\left((1-t)^2 + (1+t)^2\right)$$
$$= \frac{1}{2}\left[1^2 - \cancel{2t} + t^2 + 1^2 + \cancel{2t} + t^2\right]$$
$$= \frac{1}{2}\left[2 + 2t^2\right] = 1 + t^2$$

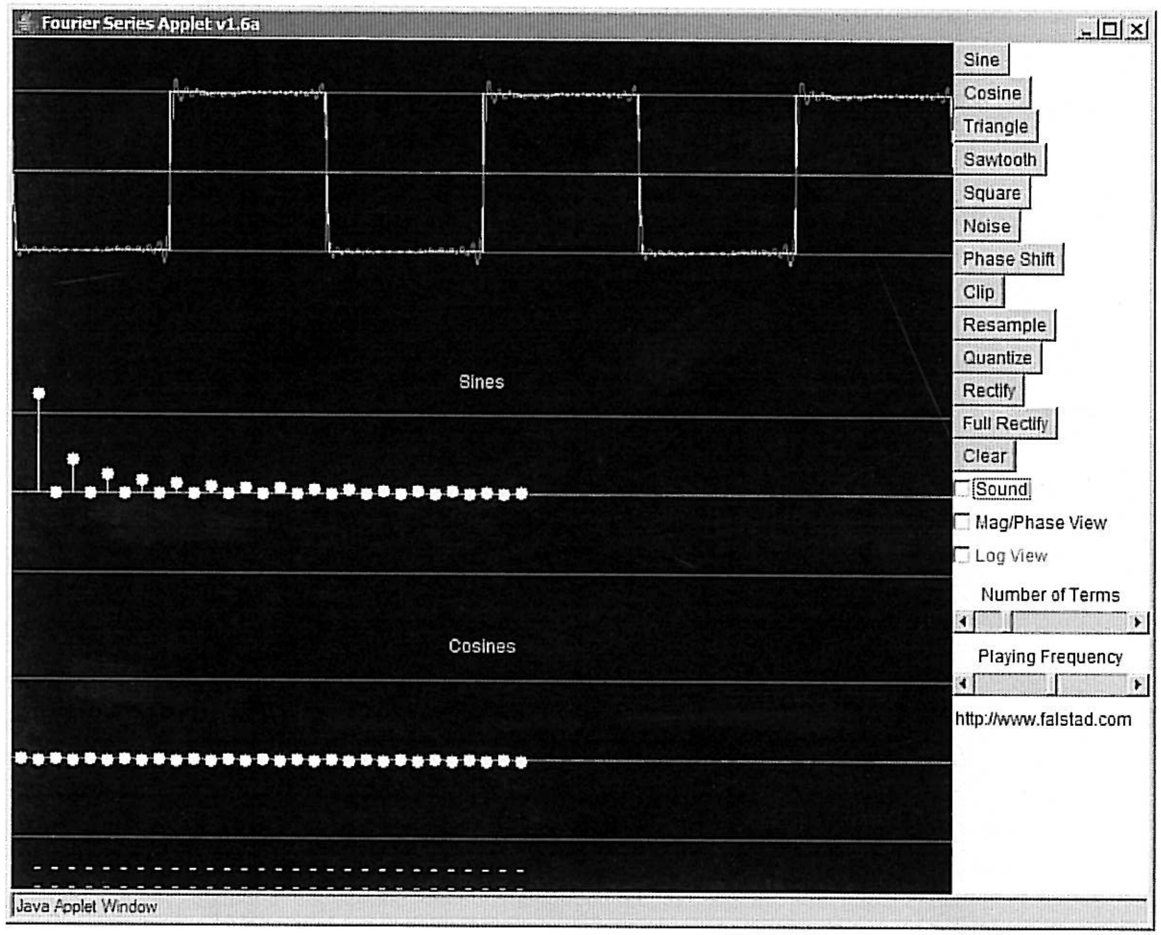
$$o(t) = \frac{1}{2}\left[g(t) - g(-t)\right]$$
$$= \frac{1}{2}\left[(1-t)^2 - (1+t)^2\right]$$
$$= \frac{1}{2}\left[\cancel{1} - 2t + \cancel{t^2} - \cancel{1} - 2t - \cancel{t^2}\right]$$
$$= \frac{1}{2} \cdot (-4t) = -2t$$

$$\boxed{\begin{aligned} e(t) &= 1 + t^2 \\ o(t) &= -2t \end{aligned}}$$

notice: e extracted the even powers (0, 2, ...)

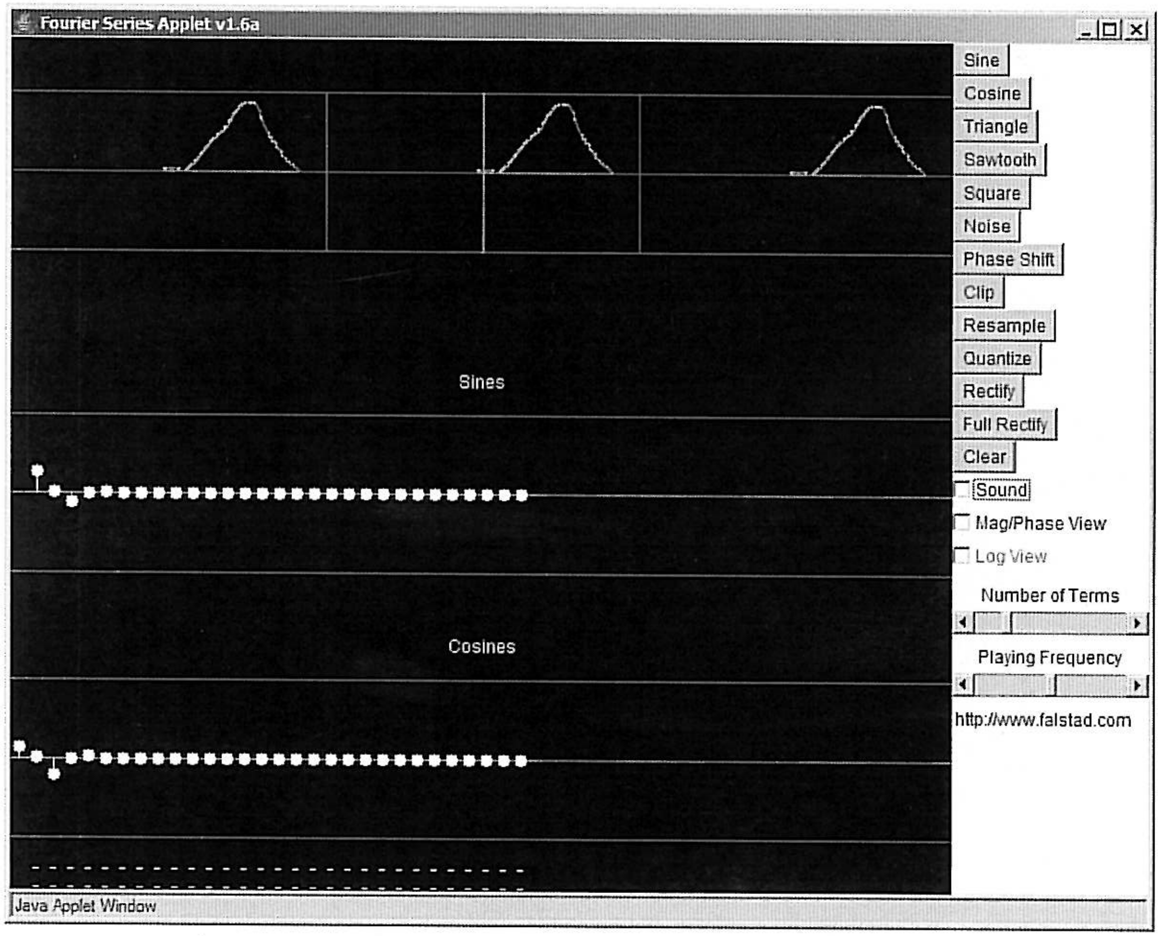
o extracted the odd ones (1, 3, ...)

3. a) 3 points
6 points



Low frequency spectral strength-- strength of high harmonics is medium (relatively high compared to part b). A sharp, abrasive timbre, sounds pretty mechanic.

b) 3 points



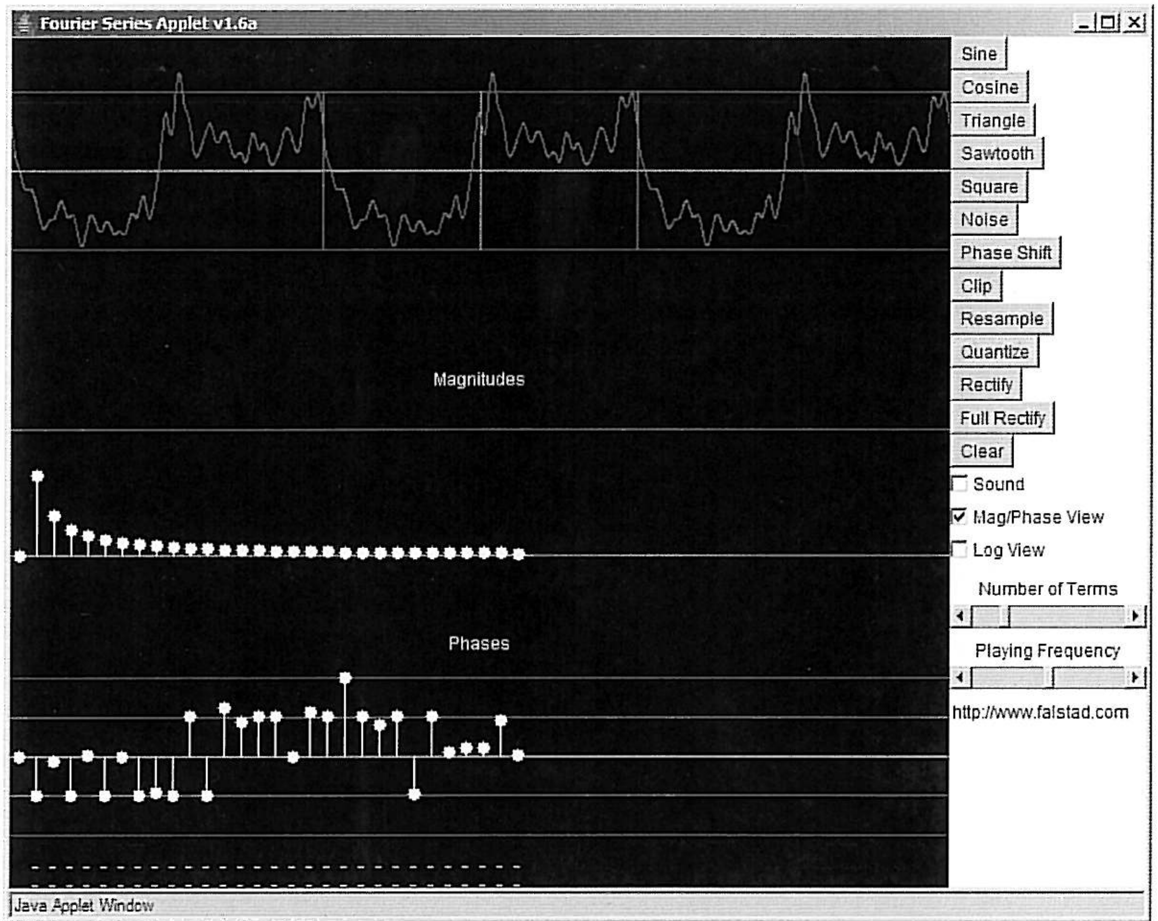
Comparatively ~~low~~ lower frequency harmonic content, little spectral power in high harmonics, much less than part (a). This sounds less abrasive, a little duller or warmer or however you'd like to describe it.

\+ Therefore, associate more abrasive sounds w/ higher harmonics,
(1 point)

#4.

4.a)
3 points
Original sawtooth

After random changes in phase



Wave form changes→ ***a lot***. *make sure you're adjusting phases of harmonics with some magnitude (power) associated with them.

b)
3 points

The timbre of the sound appears completely unaffected by phase! → ***not*** changed

5. $f = 441, 588, 735, 882$

4 points

relative fractional ratio: $1 : \frac{4}{3} : \frac{5}{3} : \frac{6}{3}$

(2)

Must be integer multiples of a "missing fundamental" for us to hear one.

Equivalent integer ratio: $3 : 4 : 5 : 6$

Harmonic numbers: 3rd, 4th, 5th, 6th

Missing fundamental = 147 Hz

This is the least-common-denominator of the frequency set, and thus represents the repetition frequency. (That's why it sounds present to our ear, though it is technically "missing.")

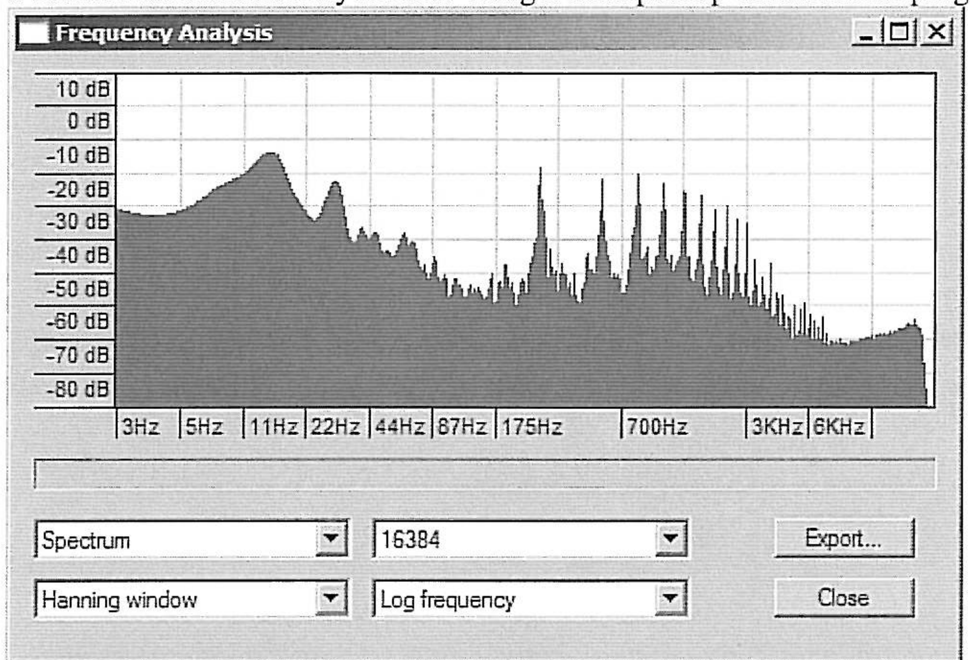
+1 point

Period = 0.00680 sec.

6. 6 points

a) 3 points

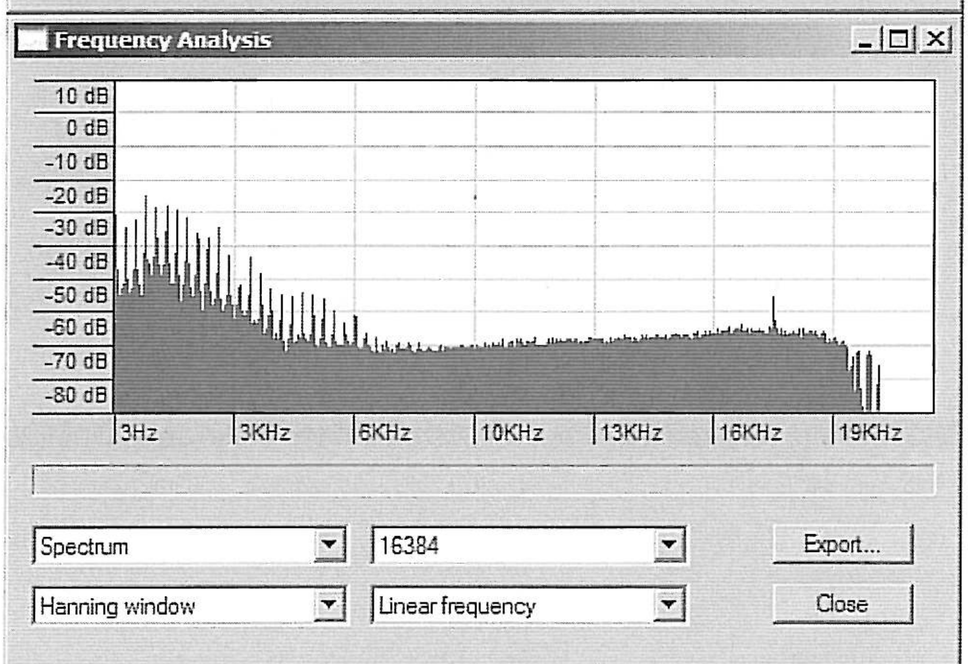
i. Partials are harmonically related. Integer multiples up the harmonic progression.



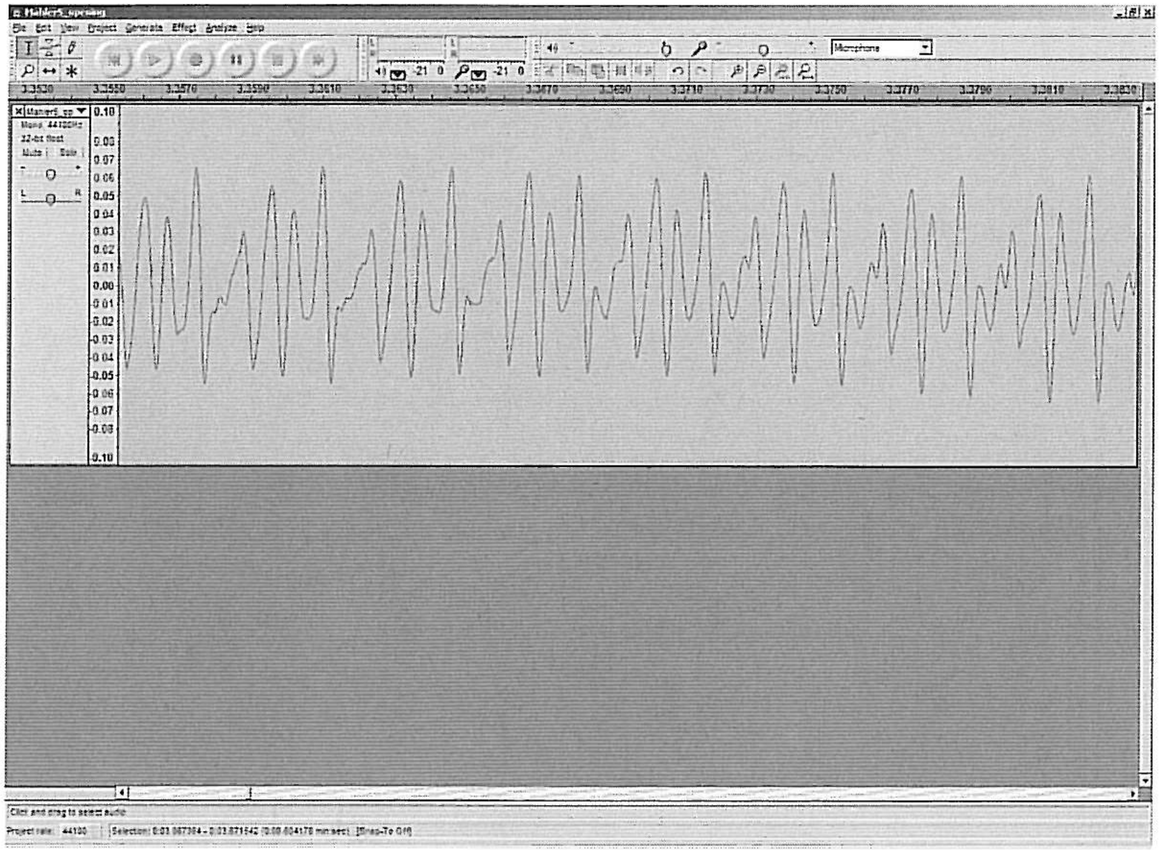
Frequencies: 284, 556, 837, 1115, 1417

Ratios: 1, 1.96, 2.94, 3.93, 4.99

within ±.1 of f, 2f, 3f, 4f, 5f etc...



ii. Time signal is close to periodic on a zoomed in time scale, though not quite:

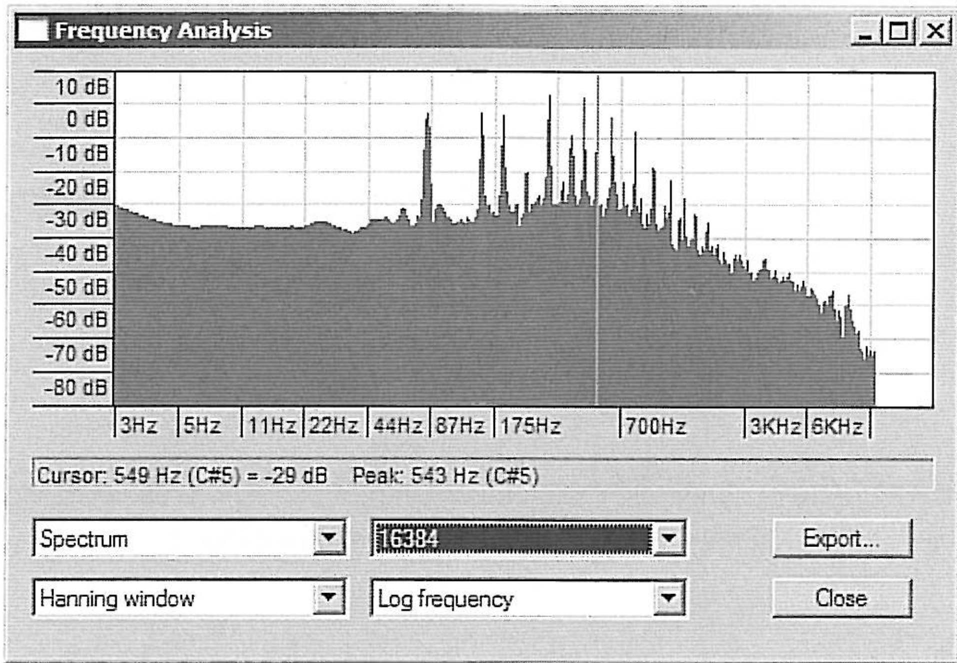


b) 3 points

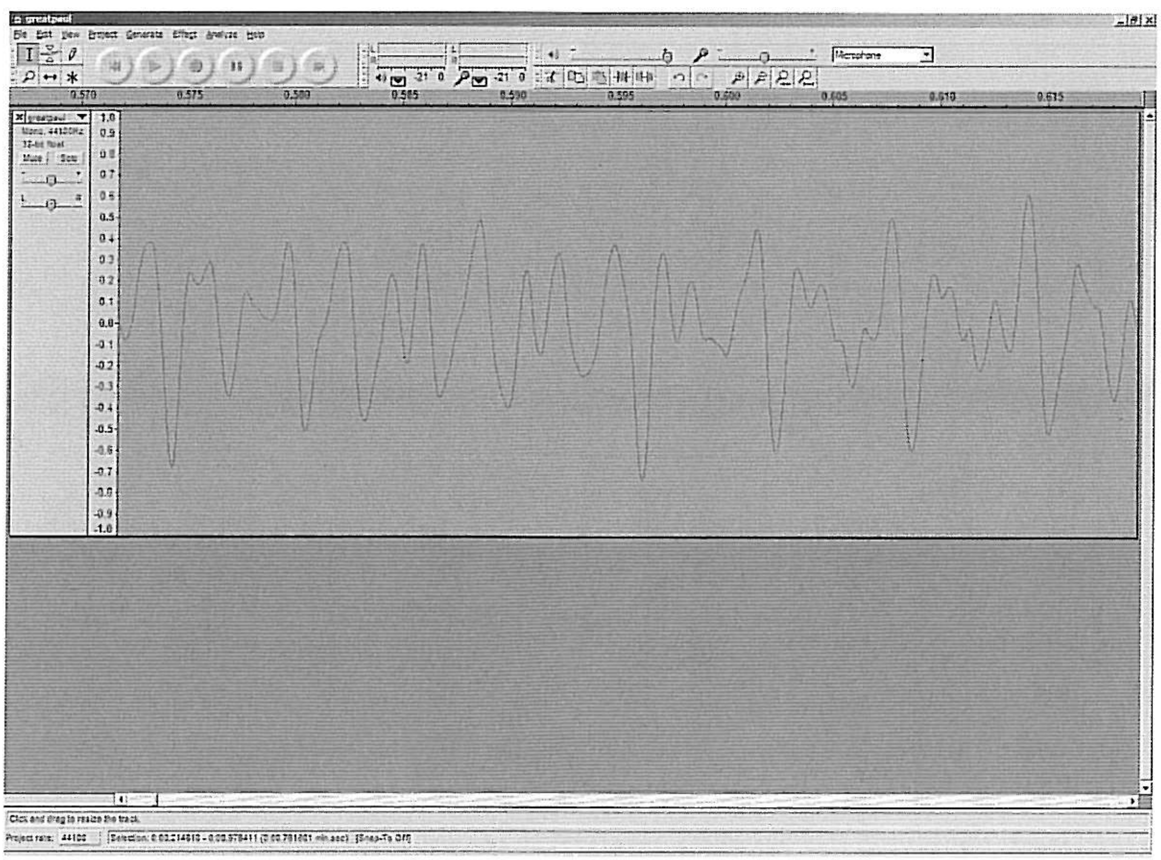
i. Partials are not harmonically related, not integer multiples of each frequency.
Frequencies observed (in Hz):

- 84
- 151
- 191
- 317
- 467 1:2
- 636
- 836

and lesser peaks. Of these lower frequency peaks, only ~~371~~ 317 → 636 forms an approximate integer multiple pair, one octave above. In general, we can see that the bell partials are not harmonically related, except for octave relations which give the overall pitch perception at the missing fundamental.

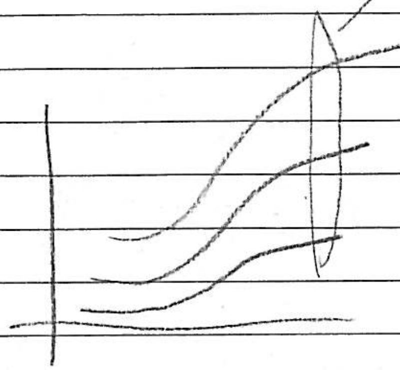


ii. Waveform is very a-periodic. Conclude that harmonic waveforms are basically periodic; non-harmonic are not periodic.



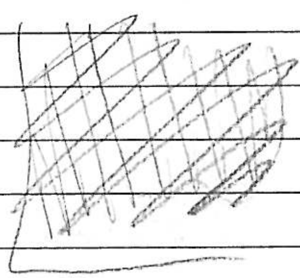
note these are all multiples of fundamental, which changes (rises).

7. a.) 6 points, 2 points



Represents a rising musical pitch (iii)

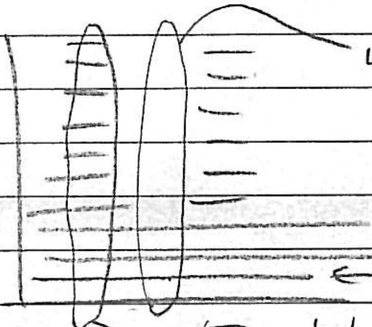
b.) 2 points



~~no particular freqs stand out.~~ ('hiss' is pitchless noise)

Represents a broadband frequency noise, such as a "hiss" (i)

c.) 2 points



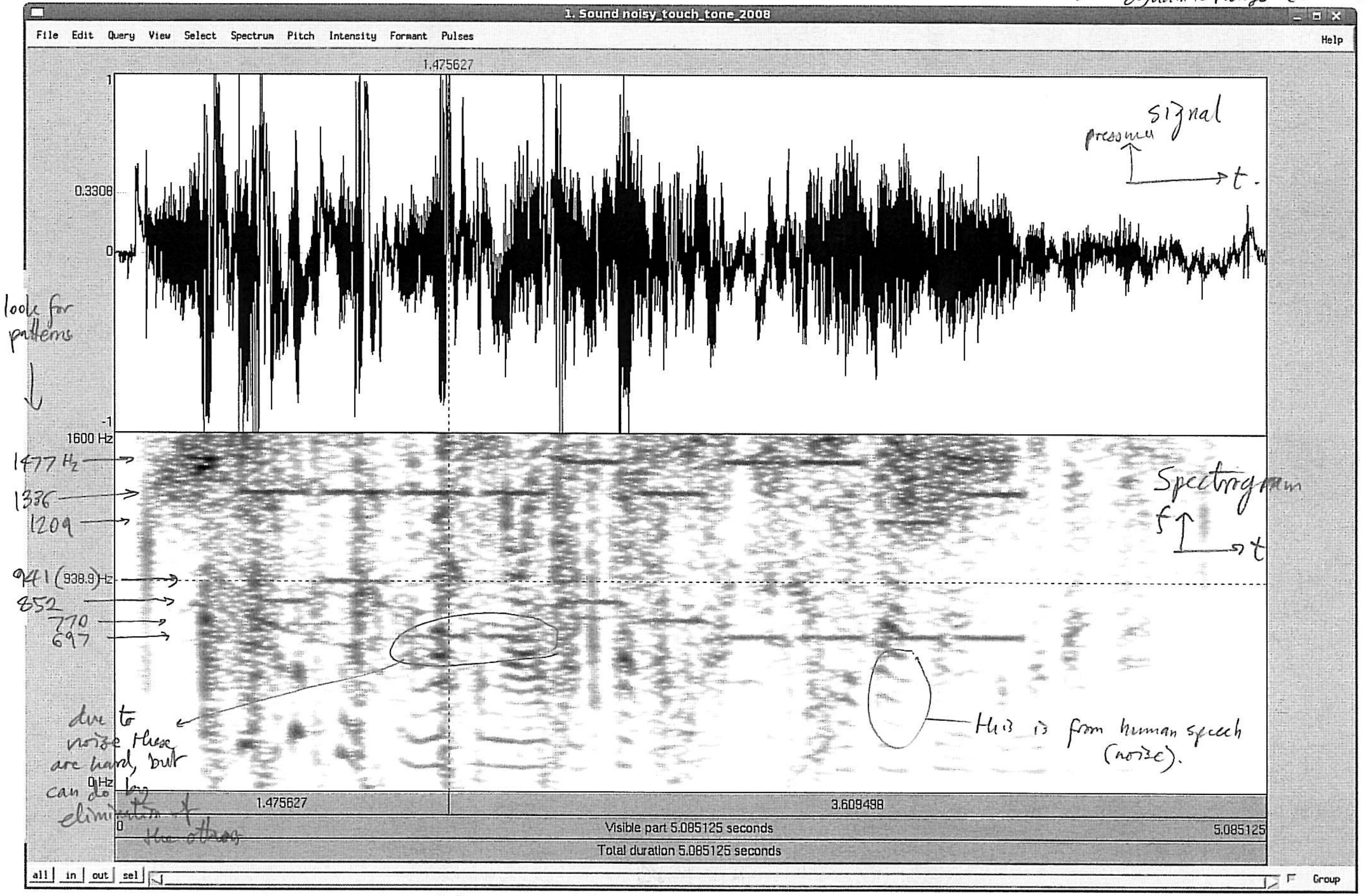
weak high harmonics ⇒ changing timbre. (but not pitch)

fundamental is constant (horizontal line)

strong high harmonics

Represents a single frequency fundamental with changes in overtone harmonics affecting timbre (ii)

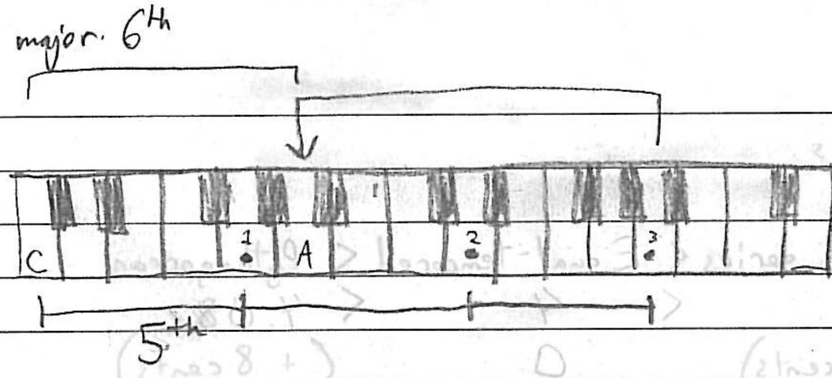
#8 (8 points) Spectrogram settings in praat: freq range 0-1600Hz, window 0.1s, dynamic range 40 or 50dB



(8 0 2) 2 9 5 - 3 3 1 2

→ Google it: Tiptop Cafe, 85 N. Main St, White River Jct, VT.

9. a)
11 points



↳ "A" an octave above original → 21 semitones away

Interval = major sixth (and octave) above

or

minor third below (and up two octaves)

b) Pythagorean ratio = $\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8} \cdot \underbrace{\frac{1}{2}}_{\text{octave}} = \frac{27}{16}$

→ Major sixth = 27/16 ratio above fundamental

Equal temperament $\quad f_{\text{fundamental}} \cdot 2^{9/12} = f_{\text{major sixth above}}$

9 semitones above C exactly.

$12 \times \frac{\ln\left(\frac{27}{16}\right)}{\ln(2)} = 9.0587$ semitones

5.87 cents sharp of 21 equal-tempered semitones

c) Pythagorean major third: $\frac{81}{64}$

↳ $12 \times \frac{\ln\left(\frac{81}{64}\right)}{\ln(2)} = 4.0782$ semitones

substantially differ by 21.5 cents

Harmonic series major third: $\frac{5}{4}$

↳ $12 \times \frac{\ln\left(\frac{5}{4}\right)}{\ln(2)} = 3.8631$ semitones

ratio
$\boxed{\frac{81}{80}}$

Major thirds

Harmonic series	<	Equal-tempered	<	Pythagorean
3.86	<	4	<	4.08
(-14 cents)		0		(+ 8 cents)