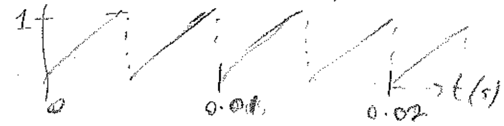
Math 5 HW1 Solutions - 2010 ① 9/29/10 Barnett

1 [9 total]

a) [2pts] $f = 200$ Hz since $\mathrm{frac}(t)$ has freq 1 Hz. (period 1 s)

$T = \frac{1}{f} = \frac{1}{200} = 0.005$ s



b) [1pt] Period is minimum, positive such time shift so signal remains unchanged.
So period is not 0.01, rather 0.005 s

c) [2pts] $\sin(\overset{\omega}{200}t)$? recall $\sin(2\pi f t)$ has freq. f so $200 = 2\pi f$

$f = \frac{200}{2\pi} \approx 31.8$ Hz

$T = \frac{1}{f} = 0.0314$ sec.

notice 3-4 digit accuracy is enough!

d) [4pts] 7 semis $= 2^{7/12} = 1.4983\ldots$

close to $3:2 = 3/2 = 1.5$

% error $= \frac{1.4983\ldots - 1.5}{1.5} \times 100\% \approx -0.113\%$

Cents error: any two numbers form a ratio, eg $r = \frac{1.4983\ldots}{1.5} \approx 0.99887\ldots$

Expressed in cents, # cents $= 1200 \frac{\log r}{\log 2} \approx -1.96$ cents

Another way to get: 3/2 is how many equal-tempered semi's? $n = 12 \frac{\log 3/2}{\log 2} \approx 7.0196\ldots$

1.96 cents more than 7.

2 [6 total]

- F6 = 1 octave & 8 semitones above A4 (440 Hz)

$f_{F6} = 440 \cdot 2 \cdot 2^{8/12}$ Hz $= 1396.91\ldots$ Hz

≈ 1397 Hz to 3 significant digits

- C4 is below A4 since C5 is 3 semi's above A4.

$f_{C4} = 440 \cdot 2^{-9/12} \approx 261.6$ Hz.

- Eb1 is 3 octaves below Eb4 which is 6 semis below A4.

so $f_{Eb1} = 440 \cdot 2^{-3} \cdot 2^{-6/12} \approx 38.9$ Hz

3. (6 pts)

a) $n = 12 \dfrac{\log \frac{422.5}{440}}{\log 2} \approx -0.703$ (70 cents flat of A4)

nearest Ab4 or G#4, 30 cents sharp of it.

you can use log to any base (eg $\log_{10}$) but I prefer you use natural log ("ln")

b) $n = 12 \dfrac{\log \frac{120}{440}}{\log 2} = -22.49$

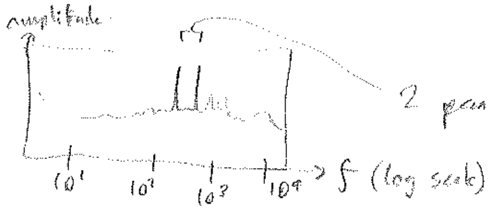
closest to $-22 = -24 + 2$

so 2 octaves below & 2 semis up from A4,

$\Rightarrow$ B2, 49 cents flat of it.

4. (6+2 bonus)

a) Spectrum



2 peaks are 393 Hz & 494 Hz

b) $n = 12 \dfrac{\log \frac{393}{440}}{\log 2} = -1.956$

$n = 12 \dfrac{\log \frac{494}{440}}{\log 2} = 2.004$

semitones relative to A4.

so 4 cents sharp of G4
0 cents sharp of B4

c) $r = \dfrac{494}{393} = 1.2570 \approx \dfrac{5}{4}$ to within 0.6%. Interval is 4 semitones, called 'major third'

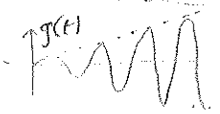
d) It's a dial tone from our landline phone system, transposed (pitch-shifted) up 2 semitones!
(usual dial tone is F4 & A4)

5. (8 pts)

a) iii)

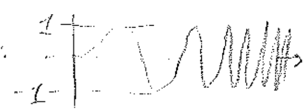
zero-crossings getting less frequent $\Rightarrow$ f decreasing.

b) iv)



$g(t) = t \sin t$ and i) does it too!

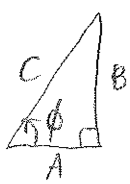
c) v) since plot it:



$\sin(t^2)$.

d) none do that.

6.



relates sin & cos ampl. to sin ampl. w/ phase ϕ.

so $A = C\cos\phi$
$B = C\sin\phi$

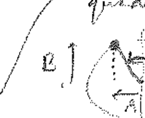
$\sin\left(200\pi t + \frac{2\pi}{3}\right)$

$C = 1$ $\quad \phi = 2\pi/3 = 120°$

so $A = 1 \cos \frac{2\pi}{3} = -\frac{1}{2}$

$B = 1 \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} \approx 0.866$

notice you're in 2nd quadrant now.



It will sound identical to $\sin(200\pi t)$ since the time offset involved ($\frac{1}{3}$ of period 0.01s) is tiny, and our ears don't care much about phase shifts. [Try out w/ audacity if you don't believe it]

[Math 5 HW1 2010]

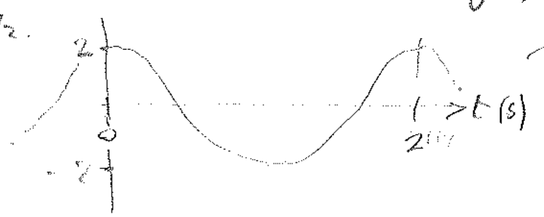
7. [3pt] a) $\sin \overbrace{400\pi t}^{\omega = 2\pi f} + \sin 402\pi t = g(t)$

[3pt] b) You hear a tone at around 200.5 Hz, changing loudness (pulsating) 1 times a second, ie beats, beating at $1\,\text{Hz} = |f_2 - f_1|$

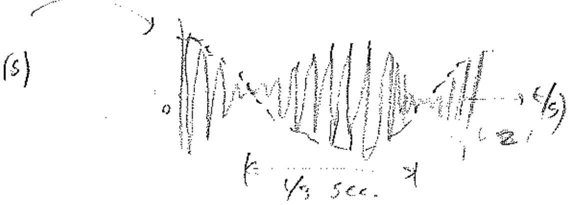
[3pt] c) using $\sin a + \sin b = 2\cos\frac{a-b}{2}\sin\frac{a+b}{2}$ (derived in handout)

and substituting $\begin{cases} a = 400\pi t \\ b = 402\pi t \end{cases}$, get $g(t) = \underbrace{2(\cos 1\pi t)}_{\text{slowly-varying amplitude}}\underbrace{(\sin 401\pi t)}_{\text{pure tone}}$

slowly-varying amplitude (from 0 to ±2), reaches largest size of values (±2) twice per 0.5 Hz cycle, ie at 1 Hz.

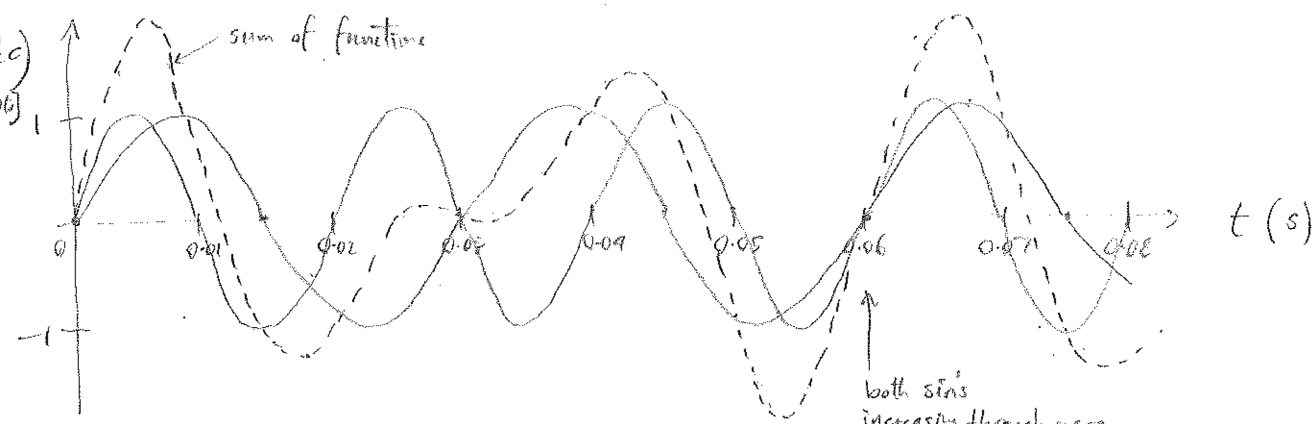


mult. by pure tone



see note on beats.

8. a) & c) [3pt] [2pt]



b) $\sin \underbrace{100\pi}_{\omega} t$ $\quad \omega = 100\pi$ so $f = \frac{\omega}{2\pi} = \frac{100\pi}{2\pi} = 50\text{Hz}$, $T = \frac{1}{50} = 0.02\,\text{s}$
[2pt]

$\sin \frac{200}{3}\pi t \quad f = \frac{\omega}{2\pi} = \frac{200\pi/3}{2\pi} = \frac{100}{3} = 33.3..\,\text{Hz} \quad T = \frac{3}{100} = 0.03\,\text{s}$

use to figure out zero-crossings first!

d) sum function repeats only when both sin curves repeat, ie are doing the same
(4pt) thing together. This is at 0.06s, ie when one has done 3 whole periods, and other has done 2.

Notice 6 = least common multiple of 2 & 3 = lcm(2, 3)

Freq. ratio $\frac{100/2}{100/3} = 3/2$ is "perfect 5th", or v. close to 7 semitones in equal temperament.