Bailey Hoar & Grace Lempres
December 7, 2010
Math 005 Final Project

Because we are both singers, we decided that we wanted our project to experiment with the mathematics of the voice. As musicians, we' d both been frustrated with groups' inability to tune notes when singing the same pitch on the same vowel (particularly the "ooo" vowel); we were both excited to learn the mathematical reasoning behind this phenomenon. Having learned about formants in class, we decided to combine our mathematical knowledge about singing with our musical knowledge.

We began our project with this question: how does your vocal training affect the way you shape your vowels, and therefore your formants, when you sing? Given our backgrounds in singing in all-female groups, and our worries that analyzing both genders would be too complicated, we decided to test only female singers. We recorded twelve Dartmouth students, including ourselves, singing a 2-octave arpeggio; the higher singers started on A Flat 3, the lower singers on G Flat 3. Each girl sang the arpeggio on three vowels—"mee," "mah," and "moo." We then analyzed each recording using the spectrogram tool of praat; we chose to analyze the formants ourselves rather than use praat's "Show Formants" tool, which is built to analyze speech, not song.

What we noticed as we recorded surprised us. Every girl who came to sing for us changed the shape of their vowel as they ascended in pitch, no matter what their training was. When we analyzed their highest note (A Flat 5 or G Flat 5), we discovered that their was little variation in perceived vowel sound. We noted that some girls could maintain a more distinct vowel in the higher range, but when we analyzed the formants of their highest note, there was virtually no difference between the F1 and F2 values for "mee," "mah," and "moo." We did notice that girls with extensive classical training seemed to maintain their vowels better, but the formants were always the same regardless of vowel.

We were puzzled by how intuitive this phenomenon was: every girl automatically changed her vowel as she ascended in pitch. When we asked our test subjects what they thought about when singing high notes, these are some of the things we heard:

- "I create space in the back of my mouth—and I open my mouth, so the vowel has to change, or it won't sound right"
- "I open everything more"
- "I open the back of my throat as if there is a very large egg in the back"
- "I aim for the best quality of sound. The vowels are not as important (relatively) when I sing high"

Put another way, when singers ascend in pitch, they are "gradually lowering the jaw as they ascend, and/or by 'smiling' more as they ascend in pitch"[1]—they truly do open everything than can be opened.

What our test subjects do intuitively, and what they are describing above, is an important phenomenon that occurs when singing. Acousticians at the University of New South Wales reported:

> Sopranos can sing at frequencies that are rather higher than the normal val- ues for the lowest resonance of their vocal tract, but failure to use this resonance would reduce both their vocal power and homogeneity in timbre. We have directly measured the resonance frequencies of the vocal tract of sopranos during singing, and find that, towards the top of their range, they consistently increase the frequency of the lowest resonance to match that of their singing. This significantly increases the loudness and the uniformity of tone, albeit at the expense of comprehensibility[2].

Though these results were not unexpected, we were shocked at how naturally singers do this, and how widespread the tuning of formants to match resonant frequencies is. It seems to be, at least for women, a subconscious and obvious choice.

---

[1] "Sopranos: Resonance tuning and vowel changes." http://www.phys.unsw.edu.au/jw/soprane.html

[2] "Tuning of vocal tract resonance by sopranos." Joliveau, Smith, and Wolfe. *Nature* 2004. http://www.phys.unsw.edu.au/jw/reprints/SopranoNat.pdf

Our observations and results for this part of the experiment were so homogenous that we decided to explore another aspect of formants. The one thing that most surprised us is how quickly singers will compromise vowel clarity—and how unintentional this compromise is. It seems natural to all the singers we interviewed that they would change their vowel in order to get "a better sound." We then wondered what would happen if you put emphasis on the vowel rather than the sound produced—what would happen to the formants? What would happen to the sound?

To explore this, we began to record singers singing not only as they usually would but also singing the same arpeggio with as distinct a vowel as possible. We don't have as much data for this part of our project, having already recorded several singers by the time we decided to record this additional data. The data we do have is very consistent; therefore, we think it is a good representation of what would have happened had we recorded all our subjects. The results were very different than the results we got for the neutral vowel. When we asked singers to maintain a "mee" all the way through the arpeggio, they were able to maintain the usual shape of the formants that you expect for a "mee" vowel. While the F1 value of the "squeaky mee," as we called it, was the same as for the "neutral mee," the F2 value of the squeaky mee jumped up to where F3 was for the neutral mee. Where we used to see a strong band—F2—for the neutral mee, almost nothing remained on the spectrogram for the squeaky mee. Equally important to note is that for the neutral mee F1 and F2 were of equal strength, while the F2 of the squeaky mee was much stronger than the F1!

Now is a good time to transition into the qualitative analysis of the "squeaky mee." After our subjects recorded their second arpeggio, we asked them to describe how it felt and sounded to sing a distinct vowel so high in their range. Some responses:

•“That’s hard…ew!”
•“It hurt”
•“Feels…tight”
•“Really awkward and difficult”

We also noticed that the sounds we recorded sounded much more forced; the singers we recorded expressed a similar distaste for the sounds they produced. Some of them didn’t even understand why we would want them to make such a unpleasant sound in the first place; others couldn’t even comprehend that we were asking them to do such an unintuitive thing. For singers, therefore, it is the natural choice to change to a neutral vowel rather than maintain the vowel sound because the sound is more forced and sounds worse.

Why, however, does the sound sound worse? As we discussed, the strength of the very high F2 indicates that high harmonics are much more prevalent in the squeaky mee, leading to a much harsher timbre—one that is less pleasant to listen to. Because the high notes our singers were recording are in such a sensitive part of the human hearing range, this harsh timbre seems particularly unpleasant. The harsh timbre would not only be less pleasant to listen to in a solo performance, but would also be much harder to blend with in a group. Singers already, as result of small variations in their vocal tract, have a hard time blending—imagine a group of sopranos singing high notes in this fashion. The overwhelming high harmonics would no doubt make it impossible to produce a pleasant, “blended” sound—or any blended sound, for that matter.

Our spectrograms also displayed how physically difficult it was to produce this sound. All of our data is only on the “mee” vowel because it was impossible to maintain a “moo” that high in one’s range. We noticed that, at the beginning of the highest note, there is a spike in the spectrogram as each singer attempted to maintain the vowel in their high register. We also noticed, after analyzing the formants, how great the variability in pitch was for the “squeaky mee.” Not only was it harder for the singer to maintain a consistent pitch when singing in this way, but the variation in pitch across the subjects was astounding. Whereas with the neutral vowel singers seemed to agree on a frequency value, the "squeaky mee" generated a large variability in pitch. This variability again contributes to the unpleasant nature of the sound and again would make the note harder to tune in a group. What's more, when singing scales or mellismas, proper pitch—so integral to the phrase—would be incredibly hard to maintain. When a singer can't agree on the pitch of one note alone, tuning half and whole steps would be even more difficult.

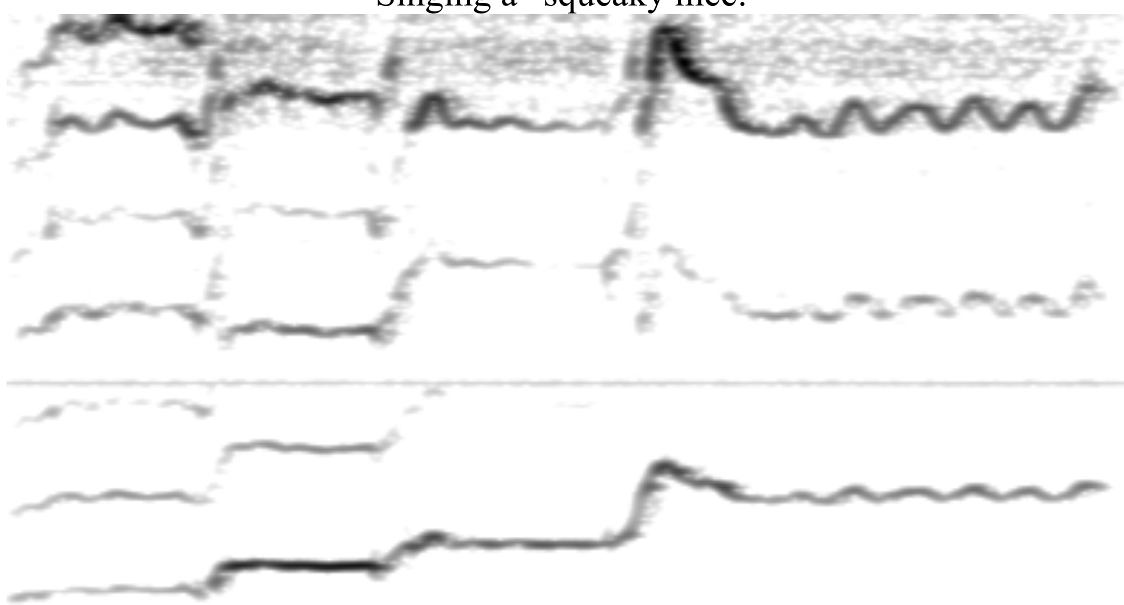
After analyzing the various problems with singing a "squeaky mee," we compared our neutral mee to one other set of data: the F1 and F2 values of "mee," "mah," and "moo" for the note an octave below (A Flat 4 or G Flat 4). As expected, we noticed a great difference between the F1 and F2 values for "mee" and the F1 and F2 values for "mah" and "moo." What we didn't expect to see was how close our average F1 and F2 values were for "mah" and "moo." While these two vowels are certainly closer in shape than they are to a "mee" vowel, we still expected them to have disctinct F1 and F2 values. We did notice a wider range of F1 and F2 values for "moo." As singers, we know how hard singers work to form their "moo" vowel, as well as how different "ooo" vowels can sound. Here, we discovered that training did play a part in the F1 and F2 values: classically-trained singers had formants that were more dissimilar to the "mah" values, whereas girls with either no formal training or musical theater training had formant values that were very similar for "moo" and "mah." For example, Anna P has no formal training, whereas Jen is a classical singer; Aislinn was trained in musical theater whereas Grace was trained classically.

Finally, we had an answer to our question: training *can* affect formant vowels, but only in the low and middle parts of a singer's voice. Once singers hit their high register, however, they naturally change their vowel in order to produce a better and more pleasant sound, one that is easier to sing, easier to keep in tune, and one that can resonate more easily. Maintaining the shape of the vowel in the high register results in unpleasantness that can be both noted on the spectrogram and in the data and heard aurally. No matter what your training, quality of sound is paramount, and every singer naturally does what she can to create the best sound possible.

Example Spectrograms

Changing to a "neutral mee:"

Singing a "squeaky mee:"



A Flat 5	F1 Mee	F2 Mee	F3 Mee	F1 Mah	F2 Mah	F3 Mah	F1 Moo	F2 Moo	F3 Moo
Danielle	830	1655	2500	845	1690	2540	846	1708	2570
Diana	832	1680	2540	830	1665	2490	832	1655	2515
Hannah	832	1680	2528	819	1667	2502	819	1665	2515
Anna F	819	1627	2436	832	1640	2475	819	1640	2490
Sara	822	1716	2558	835	1677	2532	822	1677	2545
Bailey	848	1690	2558	840	1665	2540	834	1650	2520
Grace	830	1650	2545	824	1676	2496	832	1665	2525
Aislinn	809	1638	2506	835	1664	2506	822	1651	2506
High Avg	827.75	1241.25	2521.375	832.5	1668	2510.125	828.25	1663.875	2523.25
G Flat 5	**F1 Mee**	**F2 Mee**	**F3 Mee**	**F1 Mah**	**F2 Mah**	**F3 Mah**	**F1 Moo**	**F2 Moo**	**F3 Moo**
Anna P	745	1470	2208	732	1483	2247	732	1496	2260
Jen	719	1457	2182	745	1522	2312	732	1483	2260
Annalea	705	1423	2176	706	1431	2182	719	1444	2156
Zana	719	1470	2221	706	1509	2324	758	1535	2286
Low Avg	722	1455	2196.75	722.25	1486.25	2266.25	735.25	1489.5	2240.5

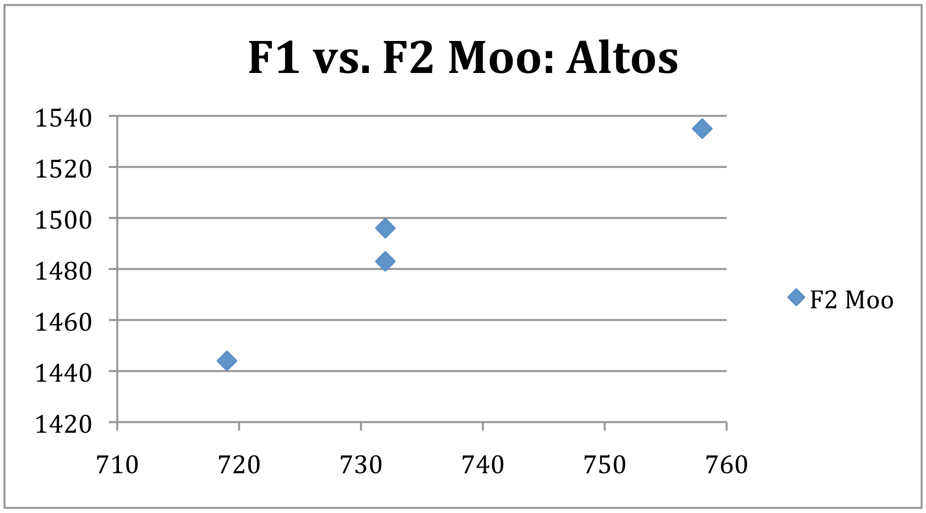
A Flat 4	F1 Mee	F2 Mee	F1 Mah	F2 Mah	F1 Moo	F2 Moo
Aislinn	400	2526	395	822	395	822
Sara	408	2519	421	822	408	835
Bailey	421	2532	422	826	420	860
Grace	416	2540	416	830	410	856

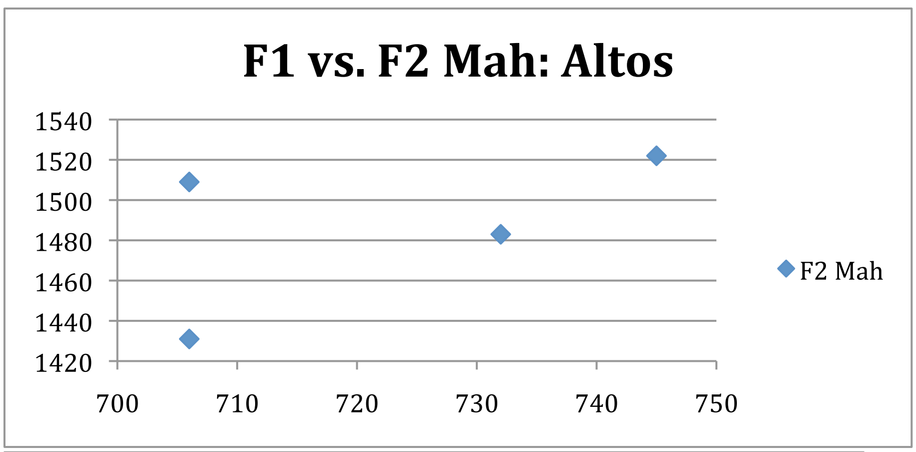
G Flat 4	F1 Mee	F2 Mee	F1 Mah	F2 Mah	F1 Moo	F2 Moo
Anna P	356	2195	356	719	356	719
Jen	369	2195	369	732	369	783
Zana	356	2221	369	758	369	770
Annalea	369	2260	369	732	369	732

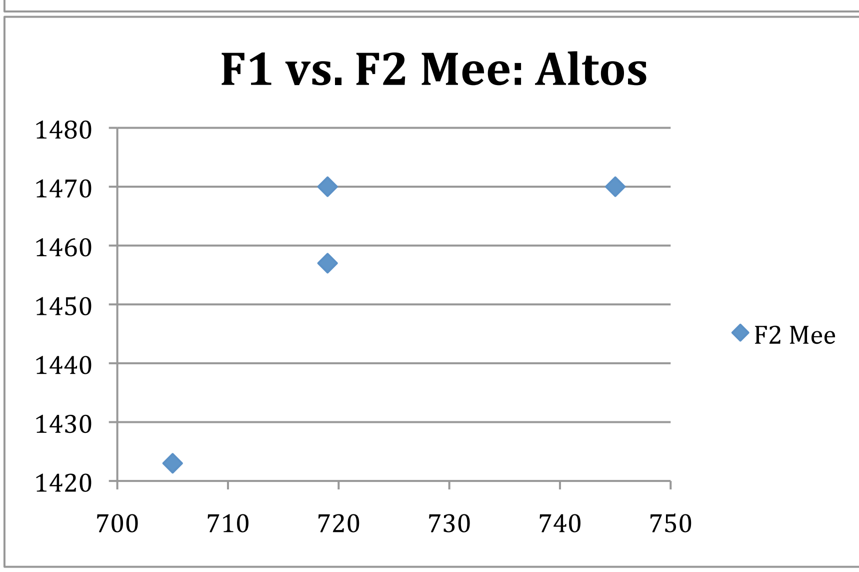
Squeaky Meee	F1	F2
Anna P	745	2234
Jen	783	2428
Zana	732	2299
Aislinn	822	2519
Bailey	835	2545

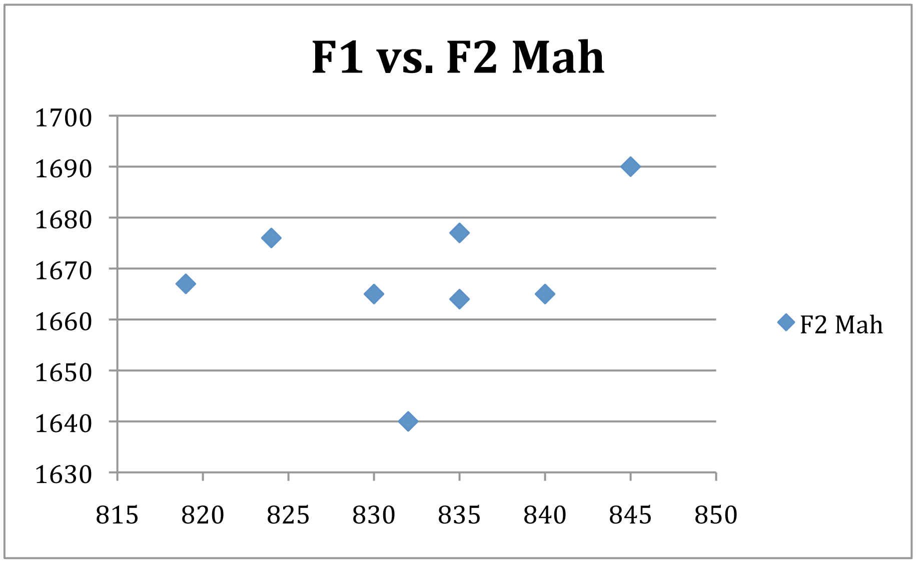
Math 5 Final Project Graphs

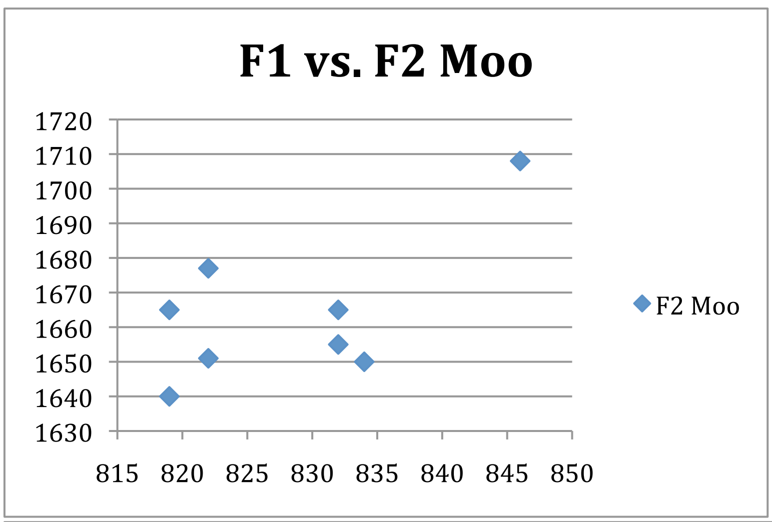
NOTE: These graphs are meant to be compared as sets. Inconsistencies within the graphs are due to variations in pitch and are less relevant for this project.

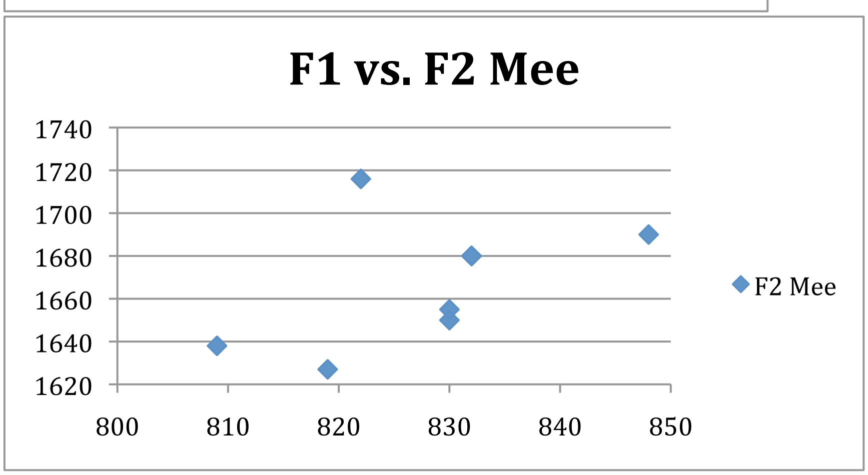







F1 vs. F2 Mah
1700
1690
1680
1670
1660
1650
1640
1630
815
820
825
830
835
840
845
850
F2 Mah


F1 vs. F2 Moo
1720
1710
1700
1690
1680
1670
1660
1650
1640
1630
815
820
825
830
835
840
845
850
F2 Moo


F1 vs. F2 Mee
1740
1720
1700
1680
1660
1640
1620
800
810
820
830
840
850
F2 Mee

Average Low Formant Values (A Flat 4)