# Math 5: Music and Sound, 2008. Midterm

2 hours, 7 questions, 60 points total

Please show working and heed the indicated number of points per question. Useful info on last page.

1. [9 points] Humans and other animals use phase difference to locate the direction a sound is coming from. Say a pure tone comes from straight ahead, and you twist your head 90 degrees so that your left ear faces the sound and your right ear faces away. The distance between your ears is 0.2 m (ignore the complication that sound has to travel around the side of your head).

   (a) What is the time delay between signals arriving at your two ears?

   (b) At what frequency are your ears one wavelength apart?

   (c) Say the signal at your left ear is $\sin(400\pi t)$. What is the signal at your right ear? (ignore any amplitude change).

   (d) Assume humans *cannot* detect a phase difference of less than 0.2 radians (about 11 degrees). For what range of frequencies are you potentially able to perceive direction? [Have you noticed this in real life?]

2. [7 points]

(a) Draw a spacetime diagram, labeling your axes, showing a fixed source emitting periodic pulses of sound (which travel in both directions).

(b) Now add to your diagram an observer (listener) *moving* rightwards, who starts to the right of the source. Use your diagram to explain whether they hear a frequency lower, the same as, or higher, than that of the source.

(c) How fast and in which direction would the observer have to move to hear a pitch a perfect fifth (3:2) higher than the source?

(d) BONUS: If you replaced the moving observer by a moving reflective wall, sketch on your diagram the new pulses and use this to say something (even a formula?) about the frequency the emitter would hear after reflection off the wall.

3. [8 points]

(a) An instrument produces a sound whose spectrum contains the following partials (measured in Hz): 400, 600, 800, 1200, 1400. What is the (likely) perceived pitch and why?

Is the signal periodic? If so, give the period. If not, explain why.

(b) A different instrument produces instead: 200, 340, 510, 680, 1020, 1046, 1637. What is the (likely) perceived pitch and why?

Is the signal periodic? If so, give the period. If not, explain why.

(c) Describe briefly the difference in sound between the two (or give examples of instruments which they could be).

4. [9 points] Sketch spectrograms on the axes provided which could realistically match the following descriptions. Feel free to highlight any features in words too:

(a) A rising musical scale played by a pure tone instrument, followed by a clap.


(b) A singer singing a fixed pitch while changing from a mellow to a harsh timbre, followed by a struck bell of no definite pitch.


(c) Say you wanted to use a spectrogram to measure the frequency of a partial to an accuracy of 2 Hz (*i.e.* quite accurate). What restrictions on the *time window* would this place?

BONUS: What types of sounds would the spectrogram then be ill-adapted to analyze?

5. [10 points] A flute player plays a single note which can be approximated by a pure sinusoid at 1575 Hz (in this question ignore vibrato or other real-world musical complications).

   (a) What equal-tempered musical pitch (give name and octave number) is this nearest, and what is the difference from this pitch in semitones?

   (b) The flute player radiates 0.01 W acoustic power equally in all directions. What intensity in $W/m^2$ would a listener hear at a distance of 30 m?

   (c) The flute player now plays much quieter so that the intensity at that distance becomes 30 dB less. To what new distance from the player would the listener need to move their chair in order to hear the same loudness as they did before?

   (d) A second flute player joins the first, and you hear a single note whose amplitude maxima pulsate at 5 Hz. From this, what can you state for certain about the frequency of the second player? (This is how musicians often 'tune up' their instruments).

6. [7 points]

(a) Construct the frequency ratio from C to the A above it in the Pythagorean C major (diatonic) scale. (Briefly show working)

(b) By how many cents does this interval differ from the equal-tempered version of the same interval?

(c) Explain if it is possible or not to have a 12-semitone (*i.e.* Western classical) tuning system in which every perfect fourth is as the Greeks would have liked it (4:3).

7. [10 points] Random short-answer questions.

(a) Write $\sin(\omega t) + \cos(\omega t)$ as a single sinusoid giving its amplitude and phase.

(b) If mass is added to the prongs of a tuning fork so that its (effective) mass becomes three times larger (viewing the fork as a mass-spring oscillator), by what musical interval (up or down?) will the pitch change?

(c) Compute the amplitude ratio between the quietest (0 dB) and loudest (130 dB) sounds a human can comfortably hear.

(d) According to the Helmholtz theory, state briefly or show in a diagram why a perfect fifth (3/2, or 7 semitones) is less dissonant than a tritone (6 semitones). (Only consider up to the sixth harmonic of the lower note).

# Useful information

$$\omega = 2\pi f$$

$$c = f\lambda$$

$$\text{dB} = 10\log_{10}\frac{I}{10^{-12}\text{W/m}^2}$$

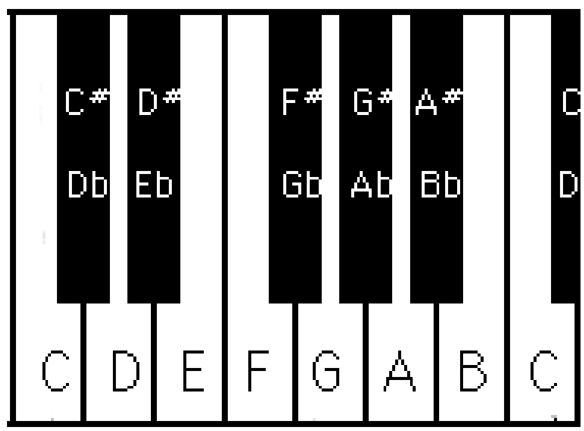
$$\frac{f_{obs}}{f} = \frac{1}{1-v/c} \quad \text{or} \quad 1+v/c$$

$$\sin(a+b) = \sin a\cos b + \cos a \sin b$$

$$\sin a + \sin b = 2\cos(\frac{a-b}{2})\sin(\frac{a+b}{2})$$

Intervals by number of semitones:

1. minor second
2. whole tone (major second)
3. minor third
4. major third
5. perfect fourth
6. tritone (augmented fourth)
7. perfect fifth
8. minor sixth
9. major sixth
10. minor seventh
11. major seventh
12. octave




The standard musical pitch A4 is 440 Hz

You can use the speed of sound as 340 m/s.