Parnett
12/9/08.

~ SOLUTIONS ~

Math 5: Music and Sound FALL 2008: Final

This was quite a hard exam:

A/B boundary 68/80

B/C " 55/80

C/D " 42/80

3 hours, 9 questions, 80 points total

Try to show working. Heed the points available for each question. Try the bonuses once the rest is ok. The last page has useful information. Good luck, have fun, and it was great to have you in the course!

1. [9 points]

2 (a) What is the frequency of the pitch C8 (the highest note on the piano) in the equal-tempered system?

C5 is 3 semitones above A4 (440 Hz)

$f_{C5} = 2^{3/12} \cdot 440$ then go up 3 octaves

$f_{C8} = 2^3 \cdot 2^{3/12} \cdot 440 = 4186$ Hz

3 (b) The second column of the touchtone keypad is encoded by frequency 1336 Hz. What musical pitch is this nearest, and what is the error from this pitch in *cents*?

ratio to A4 is $R = \frac{1336}{440}$ $n = 12 \frac{\ln R}{\ln 2} = 19.23$

$\approx$ 1 octave + $\underbrace{7 \text{ semis.}}_{\text{one fifth}}$

so nearest E6, 23 cents sharp. (error above).

2 (c) Construct the frequency ratio between C and the D above it in the Pythagorean tuned C major (diatonic) scale. (Briefly show working)

1/2 down an octave

C D E F G A B C D

1 9/8 3/2 Pythag. 3/2 Pythag.

ratio is 9:8.

2 (d) If you continued using this Pythagorean construction to compute *all* notes of the chromatic scale, what error in cents would occur when you eventually returned to (and compared against) your starting note?

$(3/2)^{12} = 129.75$, is close to 7 octaves $= 2^7 = 128$

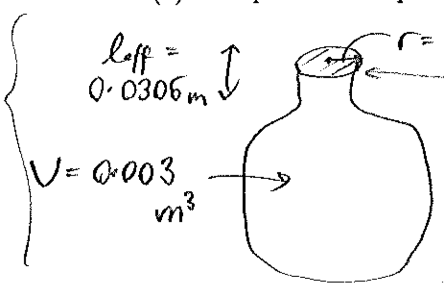
cents error $= 1200 \frac{\ln \frac{129.75}{128}}{\ln 2}$ 1 $= 23.5$ too sharp when reach C again.

you were told $1\,L = 0.001\,m^3$

2. [10 points] An empty 3 $\boxed{\text{liter}}$ soda bottle has a neck with radius 1 cm and effective length 3.06 cm.

   (a) Compute the frequency that sounds when someone (*e.g.* Mike Wu) blows across the bottle.

3

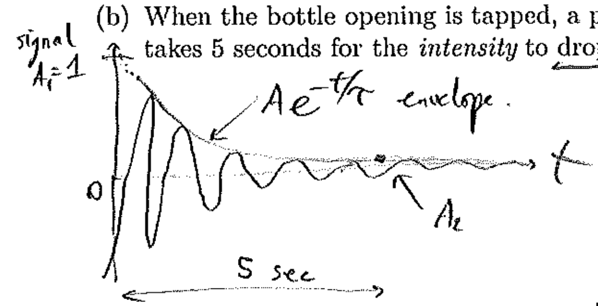


$$f_{Helm} = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}}$$

$$= \frac{340}{2\pi}\sqrt{\frac{\pi(0.01)^2}{(0.003)\,0.0306}} \approx 100.1\ Hz$$

   (b) When the bottle opening is tapped, a pressure signal of the form $Ae^{-t/\tau}\sin 2\pi ft$ is produced. It takes 5 seconds for the *intensity* to drop by 60 dB. Compute the value of the decay time. τ

3



$$\frac{I_2}{I_1} = 10^{\frac{-60}{10}} = 10^{-6} \qquad \text{so} \quad \frac{A_2}{A_1} = \sqrt{10^{-6}} = 10^{-3}$$

$$\text{so} \quad e^{-t/\tau} = 10^{-3} \quad \text{for } t=5$$

$$\text{so} \quad -5/\tau = \log(10^{-3}) \quad \Rightarrow \tau = \frac{-5}{\log(10^{-3})} = 0.72\,s$$

   (c) Someone now excites the bottle by sounding a pure tone. What *range* of frequencies would cause at least half the maximum response amplitude inside the bottle? (If you didn't get parts a and/or b, give your answer in symbols.) Sketch a response curve to illustrate this, labeling your axes.

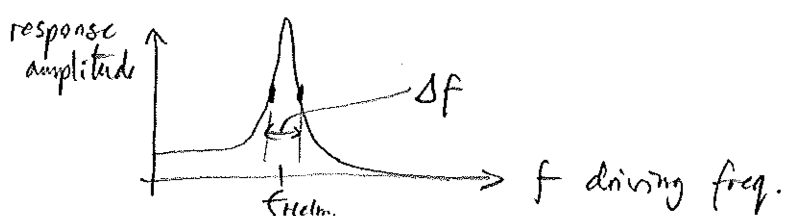
2

$$Q \text{ factor} = \pi f_{Helm}\tau = \pi(100)\cdot 0.72 \approx 227$$

(this is probably unrealistically large for a soda bottle)

$$\Delta f = \frac{f_0}{Q} \approx 0.44\ Hz, \quad \text{so range is } f_{Helm} \pm \frac{\Delta f}{2}$$

$$= [99.78,\ 100.22]\ Hz$$



   (d) How much water should be poured into the bottle to change the pitch by one octave? (does it go up or down?)

2

Pitch goes up since f_{Helm} goes up, since V decreases.

if a, l, c constant then $f_{Helm} = \frac{\text{const}}{\sqrt{V}} = \text{const}\cdot V^{-1/2}$

so $f_{Helm} \longrightarrow 2 f_{Helm}$ requires that $V \longrightarrow \frac{1}{4}V$ by ratios.

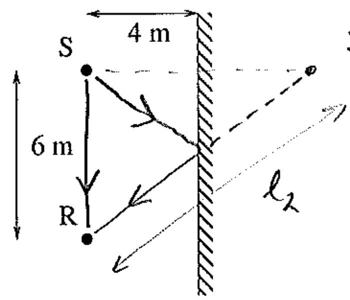
So, $\frac{3}{4}V$ must be displaced by water, ie 2.25 liters of water.

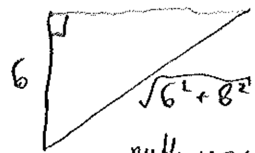
3. [10 points] A speaker cabinet S and listener R are both 4 m from a wall, and 6 m apart, as shown in plan view below.

4 (a) Sketch the direct and reflected sound paths from S to R and compute their travel times.

direct $\ell_1 = 6$ m

reflected ℓ_2 = hypotenuse drawn = 10m.

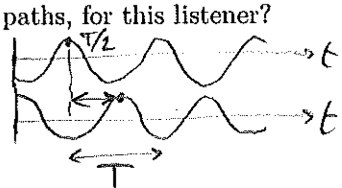




$\Rightarrow$ Travel times $t_1 = \frac{\ell_1}{c} = 0.0176$ s, $t_2 = \frac{\ell_2}{c} = 0.0294$ s.

2 (b) What is the lowest pure tone frequency emitted that would cause *destructive interference* of these two paths, for this listener?

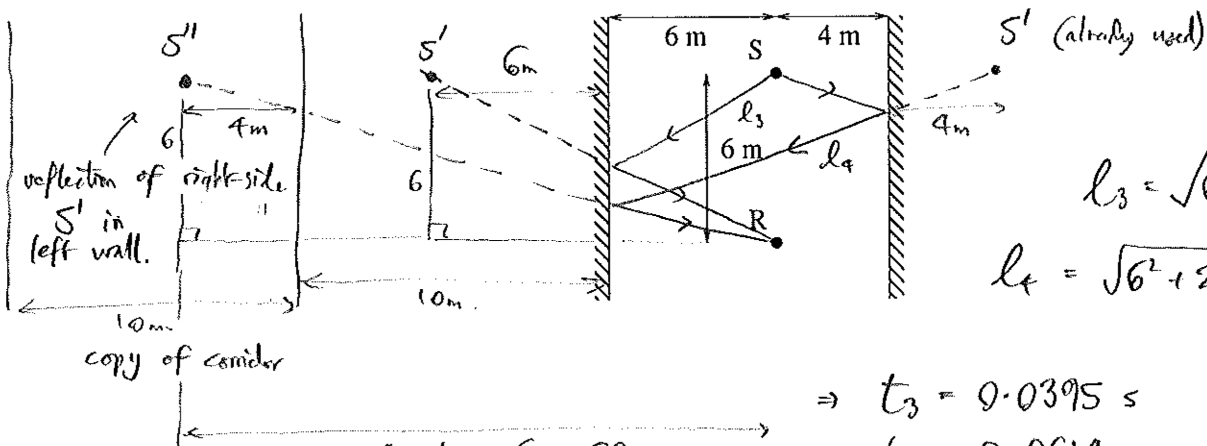
2 signals adding: destructively



requires waves to cancel, happens if delay $t_2 - t_1 = \frac{T}{2}$ where T = period. (generally $(n+\frac{1}{2})T$)

so $T = 2(t_2 - t_1) = 0.0235$, $f = \frac{1}{T} = 42.5$ Hz.

4 (c) A second parallel wall is added 10 m to the left as shown below. Find two *new* travel times which occur. Show any geometric constructions used.



$\ell_3 = \sqrt{6^2 + 12^2} = 13.4$ m

$\ell_4 = \sqrt{6^2 + 20^2} = 20.9$ m

$\Rightarrow t_3 = 0.0395$ s

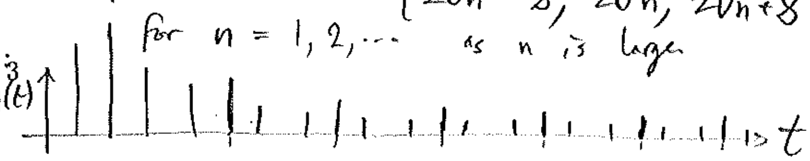
$t_4 = 0.0614$ s

(d) [BONUS] If the speaker emits a loud but short click, what would you expect the *tail* (decaying part) of the echo the listener hears to be like, and why? (diagrams help)

There is an infinite array of images S″, S‴, etc. which contribute to the tail of the echo:

In the limit the hypotenuse of triangle approaches the horizontal distance, so these path lengths approach the set $\{20n - 8, 20n, 20n + 8\}$ for $n = 1, 2, \ldots$ as n is large.

So the signal is a flutter-echo of the form:

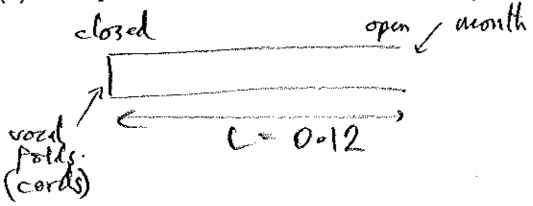


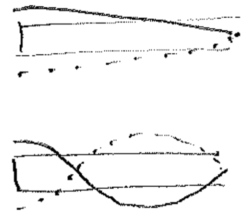
It approaches a period of $\frac{20}{340} \simeq 0.0588$ s.

4. [10 points] A child's vocal tract behaves like a closed-open pipe 12 cm long.

(a) Compute the lowest two formant frequencies assuming the tract is of uniform width.

2





$\lambda = 4L$

so $f_1 = \frac{c}{4L} = \frac{340}{0.48} = 708$ Hz

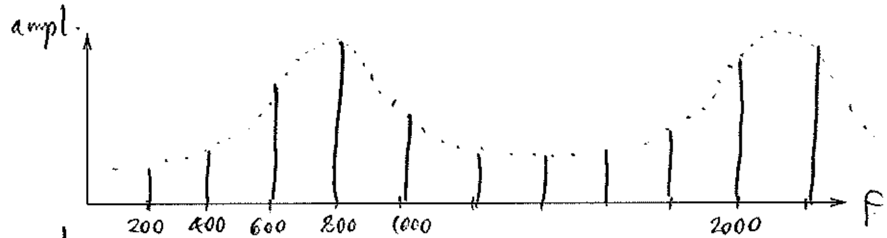
$f_2 = \frac{3c}{4L} = 2125$ Hz.

(b) Sketch graphs of a spectrum you might hear with this tract shape if the child were...

3

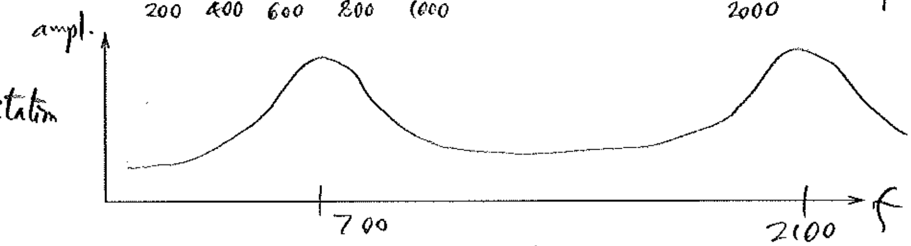
so vocal folds produce all multiples of 200Hz.

singing with pitch 200 Hz:



ie white noise excitation (pitchless)

whispering:



(be sure to label your axes and give some values on the horizontal axis)

(c) The child now breathes harmless but dense sulphur hexafluoride which halves the speed of sound in their tract. Redo the new spectrum for the first case above...

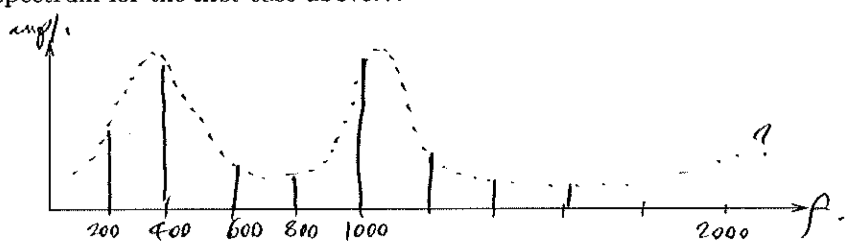
2.

formants are $\frac{c}{4L}$, $\frac{3c}{4L}$

so they are halved if c is.

$\Rightarrow f_1 = 354$, $f_2 = 1063$.

singing with pitch 200 Hz:

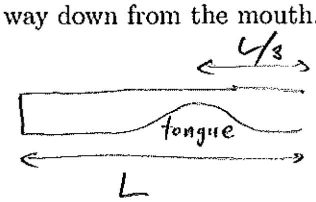


Explain what has changed and what has not: → the set of partials at $200n$, $n=1,2,\ldots$

→ the formants F1 & F2.

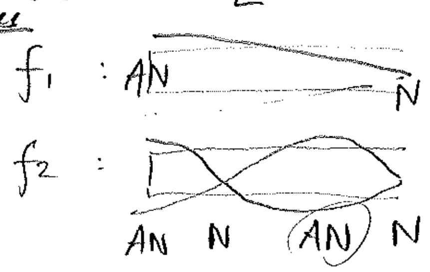
(d) Returning to normal air, the child presses their tongue upwards to constrict the tract 1/3 of the way down from the mouth. What happens to their first two formants? (relative to part a)

2.



constrict at $\left\{\begin{array}{l}\text{node}\\\text{antinode}\end{array}\right.$ ← in pressure graphs of mode $\Rightarrow$ mode freq $\left\{\begin{array}{l}\text{drops}\\\text{increases}\end{array}\right.$

Need mode shapes



slightly nearer a N, f_1 drops slightly

on an AN → f_2 increases

(e) Relative to a uniform vocal tract, how could the child easily shape their tract to lower both F1 and F2? (ie part a)

1

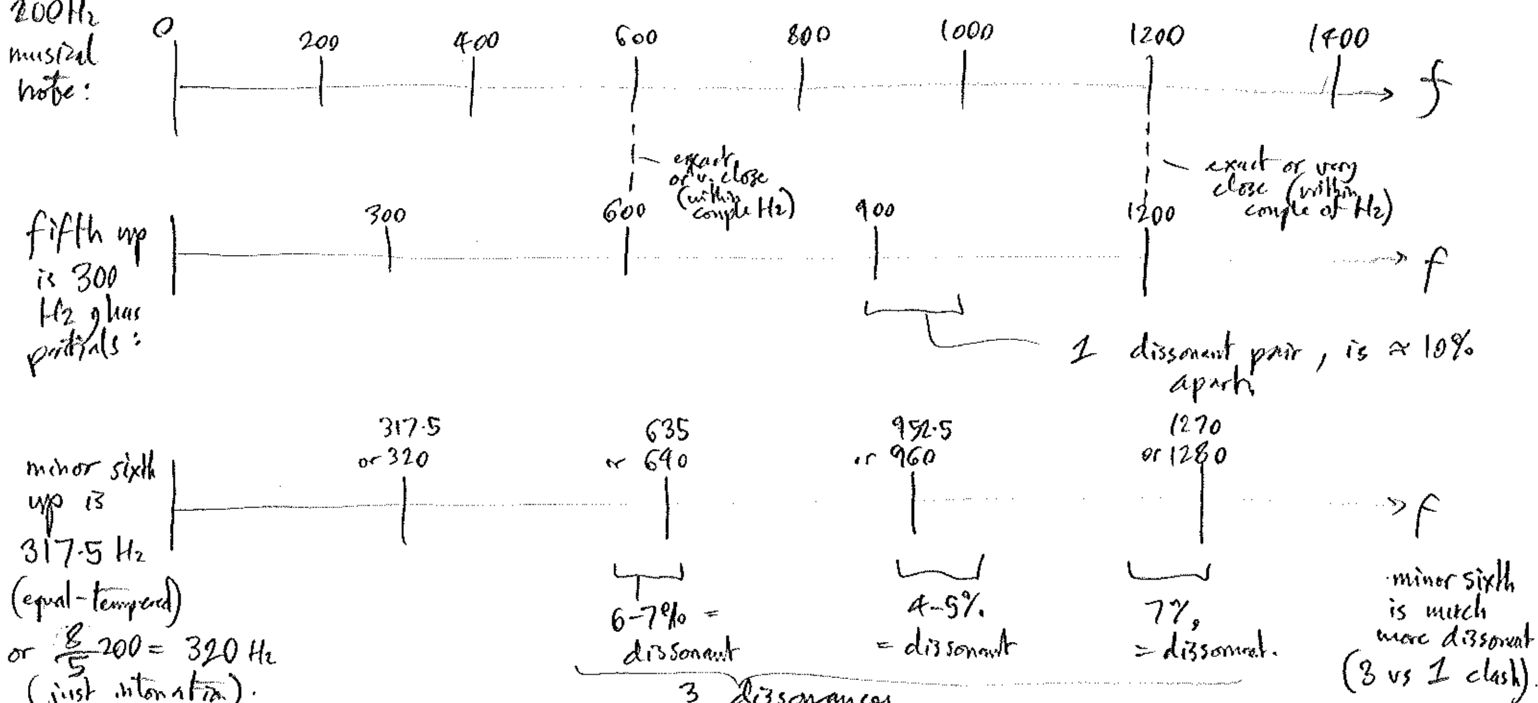
Need to constrict at a N for both modes, which occurs at the mouth.

⇒ close the mouth somewhat.

(Also correct: open the track near vocal folds, sounds harder to me!)

5. [7 points]

4 (a) A musical note with pitch 200 Hz is accomanied by another, either a perfect fifth above it, or a minor sixth above it. Compare the resulting dissonance for these two situations using Helmholtz's theory (consider only below 1400 Hz)



3 (b) What is the probable perceived pitch of a sound with partials at 432, 601, 900, 1199, 1435 Hz, and why?

ratios: 1.391, 1.498 $\approx 3/2$, 1.332 $\approx 4/3$, 1.197

$\frac{1435}{432} \approx 3.32$ no special ratio

We see
$601 \approx 2f$
$900 \approx 3f$
$1199 \approx 4f$
harmonic series

where $f = 300$ Hz is perceived pitch.

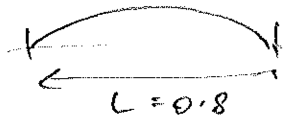
(c) [BONUS] How it can be that the piano sounds better when its octaves are tuned with ratio slightly *greater* than 2:1 ?

Each note of piano has partials which are stretched slightly (they are increasingly above nf for $n=1,2,\dots$ when n is large). Consonance requires lining up of partials so this happens when octave is tuned eg. 2.03 : 1

6. [9 points] Here you model the sound of the lowest string of the electric bass, which has length 0.8 m.

(a) The string produces a note of E1 (41.25 Hz). What is the speed of transverse waves on this string?

2



$f_1 = \dfrac{c_{string}}{2L}$ so $c_{string} = 2L f_1$

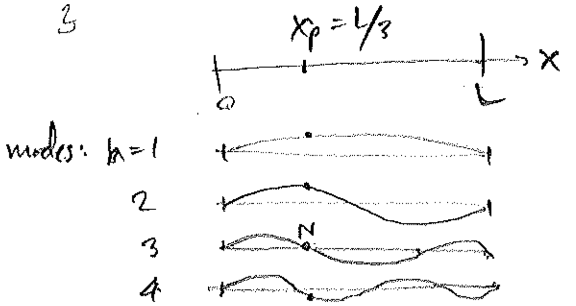
$= 2(0.8)\,41.25$

$= 66$ m/s.

↑ is perceived pitch (since $f_n = n f_1$)

(b) The string is plucked 1/3 of the way up. Compute the excitation amplitudes of the first four modes of the string (are any zero?) → yes, $\alpha_3 = 0$ since has a node where pluck.

3



excitation $\alpha_n = u_n(x_p) = \sin \frac{n\pi x_p}{L}$

$= \sin \frac{n\pi}{3}$.

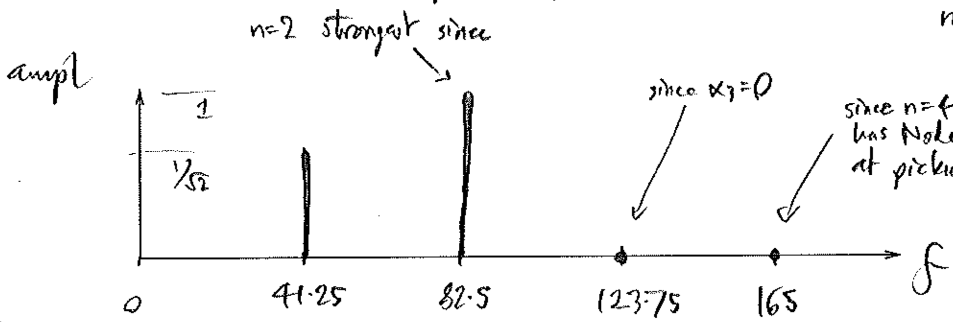
ie
$\alpha_1 = \sin \pi/3 = \frac{\sqrt{3}}{2} \approx 0.87$
$\alpha_2 = \sin 2\pi/3 = \sqrt{3}/2 \approx 0.87$
$\alpha_3 = 0$
$\alpha_4 = \sin 4\pi/3 = \sqrt{3}/2 \approx 0.87$

(c) The amplified signal is from an electric pickup placed 1/4 of the way up. Use this and part b) to compute (relative) amplitudes of the first four modes heard, and sketch the resulting spectrum.

Excitations are all equal apart from $\alpha_3 = 0$.

pickup strength depends on mode shape at $x_{pick} = L/4$.

2



ie $\sin \frac{n\pi}{4}$

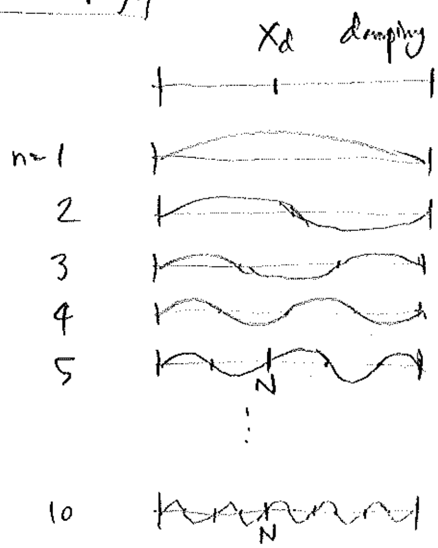
which is $\{\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0\}$ for n = 1, 2, 3, 4.

so n=2 heard $\sqrt{2}$ stronger than n=1.

the full formula here is $c_n = u_n(x_p)\,u_n(x_{pick})$.

(d) A finger is now used to damp (lightly touch) the string 2/5 of the way up. What is the pitch of the sound produced? (note this is true regardless of where it is now plucked) ↳, or where electric pickup is.

2



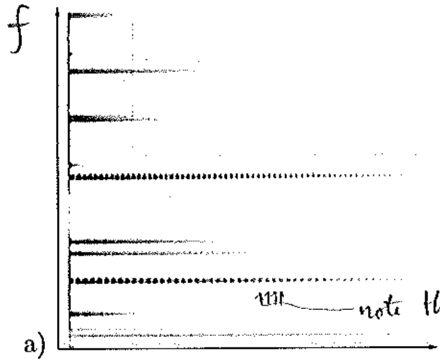
The n=5 mode is the first one with a N (node) at $x_d = \frac{2}{5}L$.

You can check that n=10, 15, etc this is also true.

⇒ spectrum contains only partials at multiples of $5f_1 \approx 206.25$ Hz

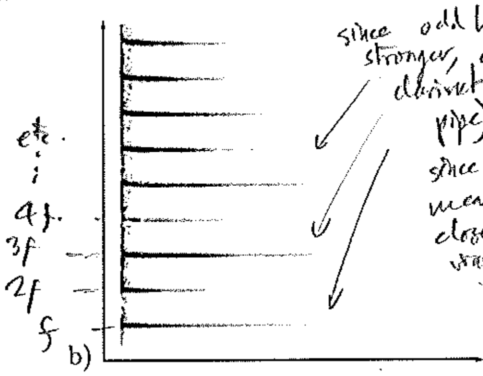
So this is the pitch heard

7. [7 points] The following show actual spectrograms, with frequency (0 to 4000 Hz) vertical, and time (0 to 3 sec) horizontal. Describe pitch (has one?), decay, timbre, and use to state an *instrument* and *method* which could have produced it: (if stuck think about the spectra)



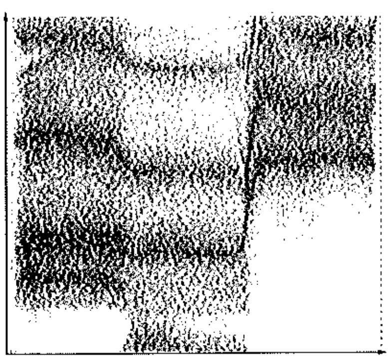
a) note the beating indicates two v. close mode freqs.

sound starts suddenly with seemingly unrelated partials, each decaying at different rates.
Suggests struck metal object, eg bell, (probably no definite pitch), or drum.
Timbre starts harsh then quickly becomes mellow.



b) since odd harmonics stronger, could be clarinet (closed-open pipe), actually since a string, means plucked close to half way along.

Also struck since partials decay, again with different decay times.
However all mode freqs appear to be multiples of some fundamental f, strong sense of pitch.
An object with these mode freqs is either a string, or pipe (it was plucked string)
Timbre again becomes mellower over time, starts harsher than a).



c) Spectrum doesn't contain partials (spikes) so it is not a freely-vibrating object.
The peaks are wide & seem to shift in time, suggesting a resonator (with 3-4 modes) changing shape, driven by white noise.
It is 3 whispered vowel sounds (showing formants F1-F4)
No definite pitch.

Which of the above, if any, will be periodic signals (at least while they last), and why?

b is the only one whose partials are all multiples of some fundamental, as is required for a periodic signal.

8. [9 points] Short unrelated calculation problems.

(a) What is the *period* of the signal $\sin(100\pi t + \pi/4)$? (as usual, t is time in seconds)

ωt so $\omega = 100\pi$

period $T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50}$ s

or 0.02 s.

(b) You are a stationary listener. How fast does a source of sound need to travel *towards* you so that its pitch appears to change by an octave?

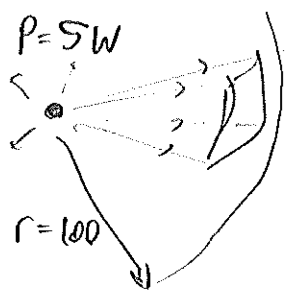
moving source Doppler formula: since coming towards, pitch goes up by octave

$f_{obs} = \frac{1}{1 - v/c} f$ but we're told $f_{obs} = 2f$ since octave up.

$\Rightarrow \frac{1}{1 - v/c} = 2 \Rightarrow 1 - \frac{v}{c} = \frac{1}{2}$ ie $v = (1 - \frac{1}{2})c = \frac{1}{2}c$

$= 170$ m/s (fast!)

(c) Compute the intensity in dB of an orchestra radiating 5 W acoustic power in all directions, at a distance of 100 m.

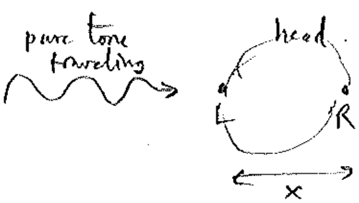


$I = \frac{P}{4\pi r^2}$ from area of sphere radius r.

$= \frac{5}{4\pi (100)^2} = 3.98 \times 10^{-5}$ W/m²

$dB = 10 \log_{10} \frac{I}{10^{-12}} = 76.0$ dB.

(d) A small animal hears a pure tone of 680 Hz. The phase difference between the signal at its left and right ears is then $\pi/5$ radians. From this find the smallest possible distance between its ears. (Ignore delays due to curvature of the head, *i.e.* assume straight-line travel of sound)



Let's assume the sound arrives along the line from L to R ear.

Then phase difference $\phi = \frac{\omega x}{c}$ where x = inter-ear distance.

(eg get this as $\phi = 2\pi \frac{x}{\lambda}$ & $\lambda =$

so $x = \frac{c\phi}{\omega} = \frac{340 \cdot \pi/5}{2\pi \cdot 680} = 0.05$ m or 5 cm.

Notice that if the incident sound comes from another direction, the ears could only be more than this distance apart. (recall worksheet on direction sensitivity).

9. [9 points] Explanation questions: points for correct and precise use of concepts. Diagrams can help.

(a) Explain the difference between frequency and pitch.

frequency : repetitions per second of some periodic signal, or some partial.

pitch : a perceived (ie psychoacoustic) frequency often due to one or many partials being 'melded' together by the ear-brain system.

(b) Explain what a Fourier series is and what kind of signals it can and cannot represent.

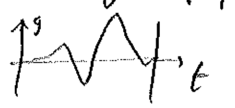
Choose a fundamental frequency f.

A Fourier series is a sum of pure tones at freq's. $f, 2f, 3f, \cdots$ etc with arbitrary amplitudes & phases.

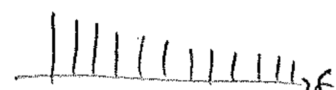
It can represent only signals periodic with period $T = \frac{1}{f}$ (or, strictly, also T/n, $n=1,2,\ldots$)

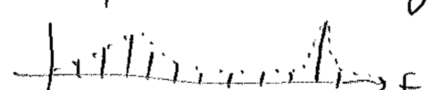
It can't represent non-periodic signals.

In terms of signal graphs over one period:

$g(t) =$  $= \alpha_1$  $+ \alpha_2$  $+ \cdots = \alpha_1 \sin(2\pi f + \phi_1) + \alpha_2 \sin(4\pi f + \phi_2) + \cdots$

(c) Explain how a Tuvan throat singer produces a high-pitched 'whistle-like' melody by singing.

The voice is a source with spectrum  which gets multiplied by the spectrum of the 'filter' which is the vocal tract resonator.

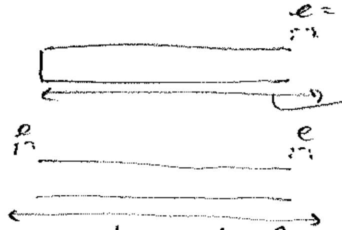
The Tuvan changes the vocal tract cavity to create a formant with a very high Q factor (eg. 20 or more), giving spectrum 

v. narrow formant

This amplifies a single partial of the source, giving heard pitch.

(d) Explain why the musical interval between a closed-open and open-open pipe of the same length is not exactly an octave (is it bigger or smaller?)

e = end corr. $\approx 0.6 \times$ radius.

c-o: o-o: 

$L_{eff} = L + e$

$L_{eff} = L + 2e$

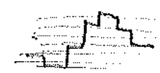
closed-open has one end correction
open-open has two " s.

so $f_{co} = \frac{c}{4(L+e)}$, $f_{oo} = \frac{c}{2(L+2e)}$ } ratio $\frac{f_{oo}}{f_{co}} = 2\frac{L+e}{L+2e} < 2$

slightly less than an octave.

(e) [BONUS] In class we learned about two ways in which digital (as opposed to analog) sound recording may change (distort) a signal in order to convert it to data. Describe one of these, and the type of distortion produced.

Either i) sampling in time may 'fold back' frequencies above $\frac{f_{sampling}}{2}$ down to lower freqs. (see worksheet on this).

or ii) quantization of signal amplitude from  to  } 2^n levels = finite number.

distorts especially for low amplitudes.