Math 5: Music and Sound. Homework 5

due Wed Oct 27

A bit shorter to let you recover from the midterm. (The last question counts towards your Aural Postings rather than HW)

1. Here we study the mathematics of how intensities of sounds 'add'. Recall that pressure signals of sounds always add, meaning if one source produces $g_1(t)$ on its own and another $g_2(t)$, then if they play together the signal is $g_1(t) + g_2(t)$. However, for intensity the story is more subtle...

   (a) Consider two *identical* signals $g_1(t) = g_2(t) = \sin 2\pi f t$ each of which we will define as having intensity I (proportional to square of amplitude). What is the intensity of the combined signal? (in terms of I) [Weird, huh?]

   (b) Now imagine $g_2(t) = \sin(2\pi f t + \theta)$, for some fixed phase θ, while g_1 is as before. For what value of θ is the combined intensity zero? [Equally weird!]

   (c) For this same case, find a formula for the combined intensity for a general fixed phase θ. [Hint: convert g_2 to a cos part, and a sin part which adds to the original. Finally convert back to "C,ϕ" format, *i.e.* , find overall amplitude]

   (d) Consider $g_1(t) = \sin 2\pi f_1 t$ and $g_2(t) = \sin 2\pi f_2 t$, now *different*, but very close, frequencies. Use the beats formula to describe how the combined intensity varies in time (what are its max and min values?)

   (e) Averaging the last result over a time much longer than the beat period $1/|f_2 - f_1|$, what is the *time-averaged* intensity? Explain why this supports our intuition that for different musicians playing together, intensities simply 'add'.

2. The signal $e^{-t/\tau} \sin(\omega t)$ describes a simple oscillator decaying after it has been struck at $t = 0$. Assume $\tau = 2$ sec.

   (a) What ω is needed so that the period is $T = 0.2$ sec?

   (b) Sketch the signal, showing the *envelope*. What is the interpretation of τ?

   (c) What is the amplitude 4 seconds after being struck?

   (d) What is the Q-factor?

3. Derive some quick 'rules of thumb' that will be useful in estimating decay times of partials:

   (a) How many dB does the intensity of *any* damped oscillator drop in one decay time τ?

   (b) How many decay times do you have to wait until it is 10 dB less than it started?

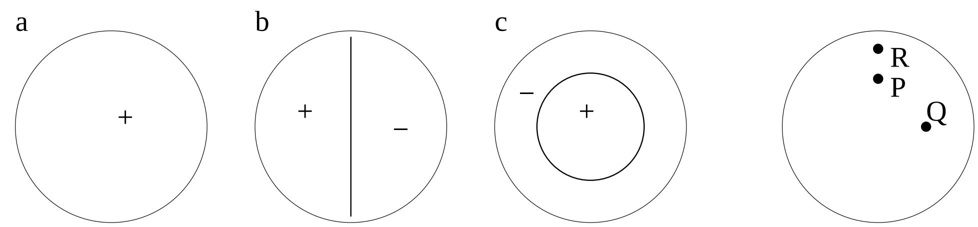
4. Estimate decay time of the damped oscillation `tuningfork_decay.ogg` recorded from a tuning fork.

   (a) Use `audacity` to decide where amplitude is down by $1/e = 0.37\cdots$

   (b) Now measure decay time more accurately by switching to `Waveform (dB)` on the pulldown menu on the track name (or using `show Intensity` in `praat`). How long does it take to get to 30 dB less intensity than it started? Compute τ from this (e.g. the slope derived in lecture).

   (c) Use your accurate decay time, and another measurement, to compute the Q factor (big, eh?)

5. Nodal lines (and the phase of the vibrations) are shown for three modes a,b,c of a circular drum, *e.g.* a tabla or conga, ascending in frequency.



If the drum is hit at the location P, describe which modes a, b, of c will be excited and which not. Repeat for Q and R. Finally, if you touch the drum at Q (damping it there), while hitting the drum *anywhere* else, which mode(s) can be excited?

6. (a) Compute the wave speed in a violin string of mass density 0.001 kg/m and tension 100 N.

   (b) What wavelength does a frequency 440 Hz have on this string? (Careful: I am not asking for the wavelength *in air*)

   (c) How long should a violin string be made so that the fundamental frequency of this string is 440 Hz (the A string of a violin)? (is this about right?)

7. Record a non-musical or musical object which demonstrates several complex vibration modes when you hit it *percussively*, and upload as an Aural Posting. You will be able to tell you have several modes because you will find several partials in the spectrogram, which usually won't be harmonically related (therefore, often a bell, clang, or drum sound). Briefly discuss partial frequencies and decay times (estimate[1] with `praat` and the results from Question 3) in your posting. Kitchens, metalwork, music shops, are good places to explore...

[1] I don't know a good way to do this when partials have a range of decay times—any of you know or develop a good way?