Math 5: Music and Sound. Homework 2

due Wed Oct 6   …but best if do relevant questions after each lecture

You will need to download the `praat` software (see our Software page) to do high-quality spectrograms. I'll show you how to use it in lecture.

For the listening questions I suggest you use *headphones* (at low volume) rather than the tiny laptop speakers. This will allow you to hear timbre much better.

1. Let's assume you hear beats only when two pure tone frequencies are 15 Hz apart or less. Compute the frequency of the highest note that produces beats when played together with the note one semitone above it. [Hint: set up an equation for f given that $f + 15$ is a semitone higher] What musical pitch is this nearest? [1]

2. In lecture 4 (9/29/10) I claimed that *any* function $g(t)$ could be written as the sum of an even and odd symmetric function. (This was needed for Fourier series). Prove this claim by:

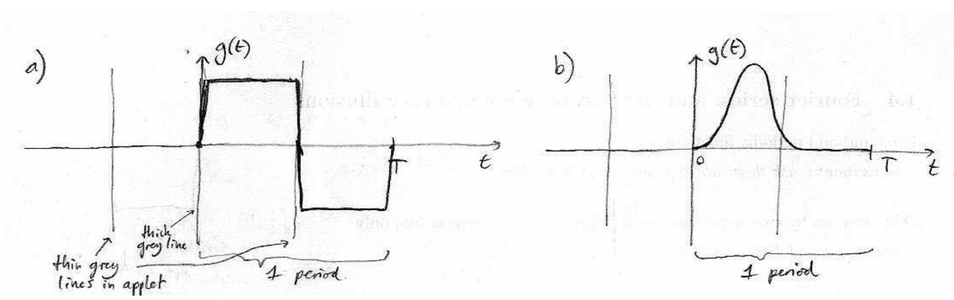
   (a) showing why $e(t) := \frac{1}{2}(g(t) + g(-t))$ is always even for any $g(t)$,

   (b) finding a similar formula for a function $o(t)$ that is always odd,

   (c) showing why adding $e(t)$ and $o(t)$ gives $g(t)$.

   (d) Tell me what $e(t)$ and $o(t)$ are for the function $g(t) = (1-t)^2$ (don't forget to expand and simplify your answer).

3. Sketch using the mouse the following two periodic graphs into the online Falstad Fourier applet, reasonably accurately. For each, comment on the harmonic content (strength of high harmonics) and the effect on the timbre you hear.



   What do you conclude about the harmonic content of discontinuous (jumpy) graphs vs smooth graphs?

4. Here we study whether the *phases* of harmonic content are important for timbre. [You may want to check up part c) of the worksheet `zerocross`]

   (a) Take a periodic signal such as sawtooth in the Falstad Fourier applet. Check `Mag/Phase View` to view 'magnitudes' (amplitude coefficients $c_1, c_2, \ldots$) and phases ($\phi_1, \phi_2, \ldots$). Adjust a bunch of phases: does the waveform change? (not, a little, a lot?)

[1] Maybe this explains why musically, semitone intervals are almost unheard-of at low pitches? They are more common (although not very) for high pitches

(b) Does the timbre change when phases are changed? (not, a little, a lot?) The best way to test this is to switch off the sound while you change the phases, then switch it on and off to compare the new timbre against the old. (If you don't switch on and off, you'll hear some *overall volume* changes and may misinterpret this).

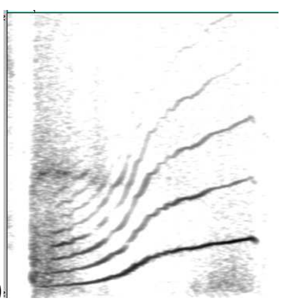
5. Compute the 'missing fundamental' frequency that you would probably) hear if pure tones at the following frequencies were played together: 441 Hz, 588 Hz, 735 Hz, and 882 Hz. State the harmonic numbers (*i.e.* 2nd, 3rd, etc) for each frequency. BONUS: what is the period of the combined signal?

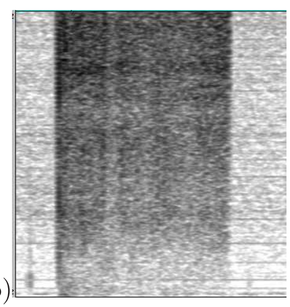
6. Measure the first few partials present in the following two sounds (only consider strong partials, not random little bumps). The sounds are on the HW page; use `audacity` spectrum tool once you've highlighted a relevant part of the sound (read hints on the Software page). For each sound, answer this: i) are the partials *harmonically* related? Zoom in on that bit of signal so you can see the oscillations: ii) how close to periodic is it?

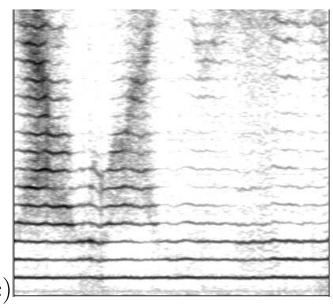
   (a) The first note the trumpet plays in 'Mahler's Fifth Symphony (trumpet opening)',

   (b) Any part of 'Great Paul bell, St Paul's Cathedral, London'.

7. For each of these three spectrograms (made with `praat`), decide which from the list (i)-(v) it could be, and explain why. [If stuck: look at spectrograms of your own human-generated sounds for comparison!] Time is horizontal, frequency vertical.

a) 

b) 

c) 

   Here are the possibilities: (i) hiss, (ii) a single musical pitch which changes timbre, (iii) a rising musical pitch, (iv) a falling musical pitch, (v) three different musical pitches played one after another.

8. The FBI has given you an audio file (Telephone touch tones, on the HW page) recorded by a hidden microphone. There is so much background noise that a human can barely hear the dialing. Using the information on touch tone codes from class, use a spectrogram (*e.g.* with `praat`) to identify the 10-digit telephone number (your CSI moment). Then Google it to find the location of the crime!

9. Basics of tuning systems.

   (a) If you started at C, climbed up by a perfect fifth (3:2) three times, then moved down an octave, what note of the diatonic scale would you get to, and what musical interval does it form with the original C?

   (b) Compare the Pythagorean ratio for the above interval with the equal-tempered equivalent, expressing the error in cents.

   (c) Compute the ratio between the major third occurring in the Pythagorean scale and that occurring naturally in the harmonic series (this ratio is the so-called *Syntonic comma*). Express it in cents. Where (in cents) does the equal-tempered major third fall relative to these two other major thirds?