# Math 5: Music and Sound. Homework 1

due Wed Sep 29 ... but best if do relevant questions after each lecture

First install audacity onto your laptop and make sure you can record a sound, play it back. See our Software page for help.

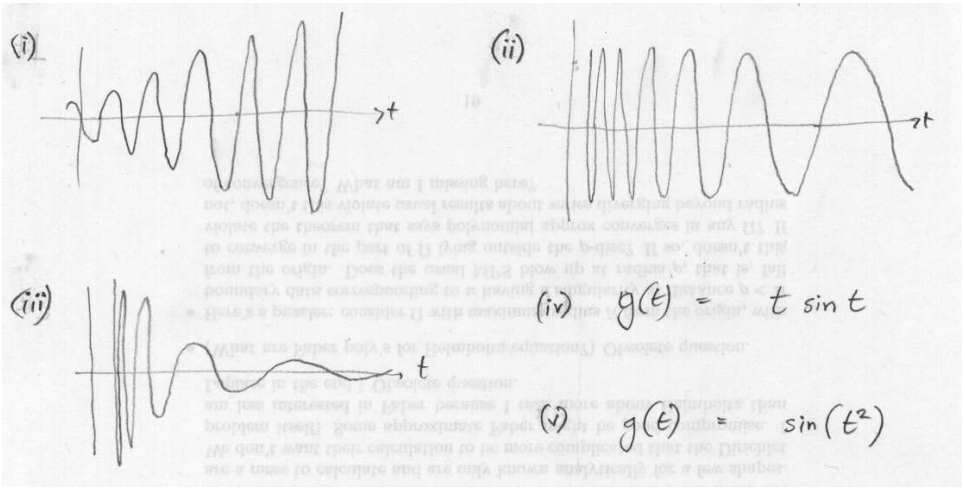
1. (a) If $t$ expresses time in seconds, what is the period of the signal $f(t) = \text{frac}(400t)$ ? (recall frac means fractional part)
   (b) For this signal it is true that $f(t + 0.01) = f(t)$ for all $t$. Explain whether this means its period must then be 0.01 sec.
   (c) What is the period, and frequency, of the signal $\sin(400t)$ ?
   (d) Compare the frequency ratio 3:2 to that given by 7 equal-tempered semitones, and express their difference both as a *percentage* error (*e.g.* 999 Hz is 0.1% flat relative to 1000 Hz), and in *cents*. [Hint: divide the two ratios to give a ratio very close to 1, work with this]

2. Given the modern A4 of 440 Hz, compute the frequencies of the following notes using equal-tempered tuning (give them to 3 significant digits): F6 (the soprano top F in Mozart's *The Magic Flute*), C4 (careful—is this below or above A4?), the famous low E♭1 that opens Wagner's *Ring Cycle*.

3. Find which (equal-tempered, modern) note names the following frequencies are nearest, and express how out of tune they are from these notes, in cents.
   (a) Handel's 'A4' tuning fork from 18th century which still survives and is tuned to 422.5 Hz (no, it didn't drift).
   (b) The 'interference' hum you might hear at 50 Hz due to an electrical system, in Europe (in the US it's 60 Hz).

4. Download *Mystery Sound 1* from the HW page.
   (a) Find the two main component frequencies as accurately as you can using `audacity`. [Hint: Analyse → Plot Spectrum, make it fill the screen, use the Log scale, and choose the longest transform length 16384. `audacity` suggests note names ... but you have no idea how far off they are].
   (b) Express them as notes in the equal-tempered system with tuning errors from these notes in *cents*.
   (c) Compute their frequency ratio, and convert it to a musical interval. The ratio is close to one involving small integers—which ones?
   (d) BONUS: Do some detective work and explain what this (becoming less) familiar sound is, and why it is not as familiar as usual...

5. Five functions, which we can interpret as pressure vs time, are given below either as a graph or a formula. State which (and there may be none, or more than one) of them are...
   (a) Decreasing in amplitude and in frequency
   (b) Getting louder but at constant pitch (ignore any subtle pitch-loudness perception issues for now!)
   (c) Increasing in pitch but not in amplitude
   (d) Decreasing amplitude but at constant frequency




6. The pressure signal $\sin(200\pi t + 2\pi/3)$ can be written as $A\sin(200\pi t) + B\cos(200\pi t)$. Find the constants $A$ and $B$. Explain with reasons whether you expect this signal to sound the same as $\sin(200\pi t)$ to the ear.

7. Two sinusoidal tones of frequencies 100 Hz and 103 Hz are played together both with amplitude 1.

   (a) Write down a formula for their combined (added) signal.

   (b) Describe what you would hear. [you might easily check this by generating then mixing two tone tracks with `audacity`, but this is not required].

   (c) Use a trig identity to re-express this signal in a form more useful for *understanding* what you hear. Explain how the two terms in this formula correspond to *aspects* of what you hear.

8. Draw a time axis labelled 0, 0.01, 0.02 etc up to 0.08 (think of this as units of seconds).

   (a) Sketch (without using a computer, since you'll want to practise for doing this in an exam) the graphs of $\sin(100\pi t)$ and $\sin(\frac{200}{3}\pi t)$ on these axes. [Hint: first get the zero-crossings right]

   (b) What are the individual frequencies, and periods, of these functions?

   (c) Add to your sketch (in a different color) an estimate of the *sum* of the functions.

   (d) What is the period of this summed function? What musical interval would you hear when the signals are played together?

9. Upload to our Aural Postings page an interesting sound (maybe recorded with audacity and saved in OGG format) illustrating a concept from the first week of class (explain *how* it illustrates it!)