10/20/08
Barnett

SOLUTIONS

Math 5: Music and Sound. Quiz 1

~ 2008.

30 mins (4 questions, Qu 3 worth less than the others)

Please write on this paper, show your working. The last page has useful information.

1. Consider the signal $2\sin(\underbrace{400\pi t}_{\omega t} + \pi/4)$ so $\omega = 400\pi$

   (a) What is the period?

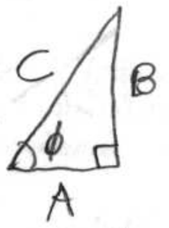
   $$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\cancel{\pi}}{400\cancel{\pi}} = \frac{1}{200}$$

2

   $$= 0.005 \text{ (seconds)}$$

   (b) Rewrite it in the form $A\sin(\omega t) + B\cos(\omega t)$. ← this means: solve for A & B! (you can use addition formula on back page)

use the triangle:



rotating vectors gives $C\sin(\omega t + \phi) = A\sin\omega t + B\cos\omega t$

where $A = C\cos\phi$
$B = C\sin\phi$

[$\omega = 400\pi$ as above].

here $C = 2$, $\phi = -\pi/4$ ($-45°$) so $\cos\phi = -\sin\phi = \frac{1}{\sqrt{2}}$

3

so $A = \frac{2}{\sqrt{2}} = \sqrt{2} \approx 1.41$, $B = -\frac{2}{\sqrt{2}} = -\sqrt{2} \approx -1.41$

2. (a) What musical pitch (give name and octave, *e.g.* D#3) is nearest the frequency 1047 Hz?

ratio $r = \frac{1109}{440}$

semitones above A4 $= 12\frac{\ln r}{\ln 2}$

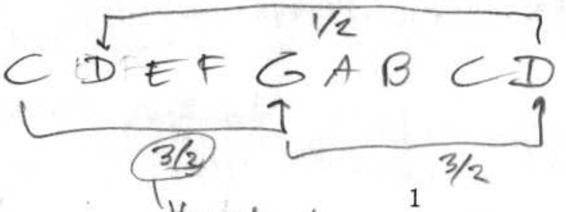
≈ 16.0

2

$= \underbrace{12}_{1\text{ octave}} + \underbrace{4}_{3\text{ semitone}}$.

so it's nearest C#6.

   (b) Find the frequency ratio of the *whole tone* interval in the Pythagorean scale.



$\left(\frac{3}{2}\right)^2 \cdot \frac{1}{2} = 9/8$

2

the interval on which Pythagorean scale (temperament) based.

2 semitones.

(c) Find the frequency ratio between this whole tone and the equal-tempered whole tone in cents

2 $9/8$ Pythag whole tone

$2^{2/12}$ equal-tempered whole tone

$R = \frac{9/8}{2^{2/12}} = 1.00226\ldots$, number of cents $c = 1200 \frac{\ln R}{\ln 2}$

$= +3.9$ cents.

since $9/8 > 2^{2/12}$ the Pythag. is sharp relative to equal tempered

3. What would you hear if two pure tones at frequencies 2000 Hz and 2008 Hz were played together? (Give any relevant new frequencies)

f_1 f_2

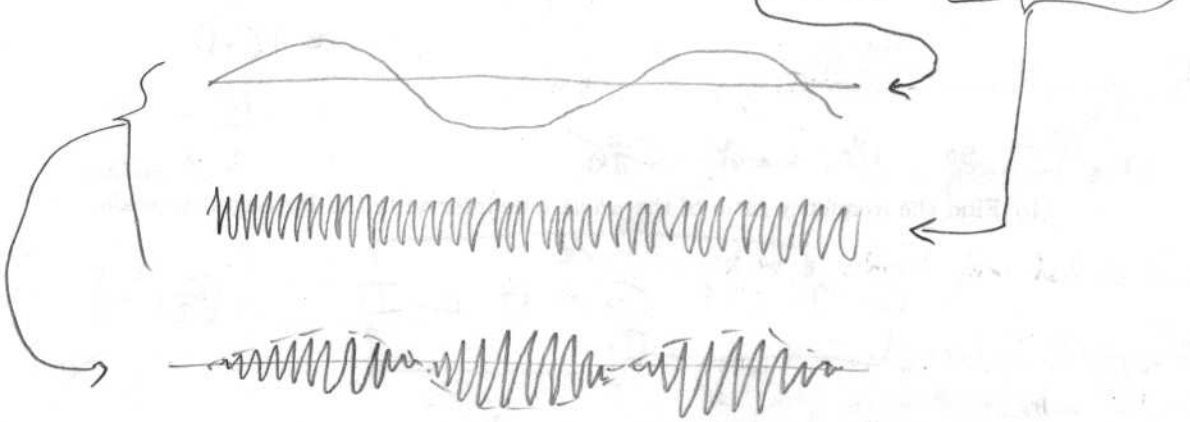
You would hear a pure tone at 2004 Hz 'modulated' (ie with amplitude changing) periodically at 8 Hz. 3

The 'wah-wah' amplitude modulation has frequency $f_1 - f_2 = 8$ Hz.

you didn't have to write this

This follows from the sum of sinusoids

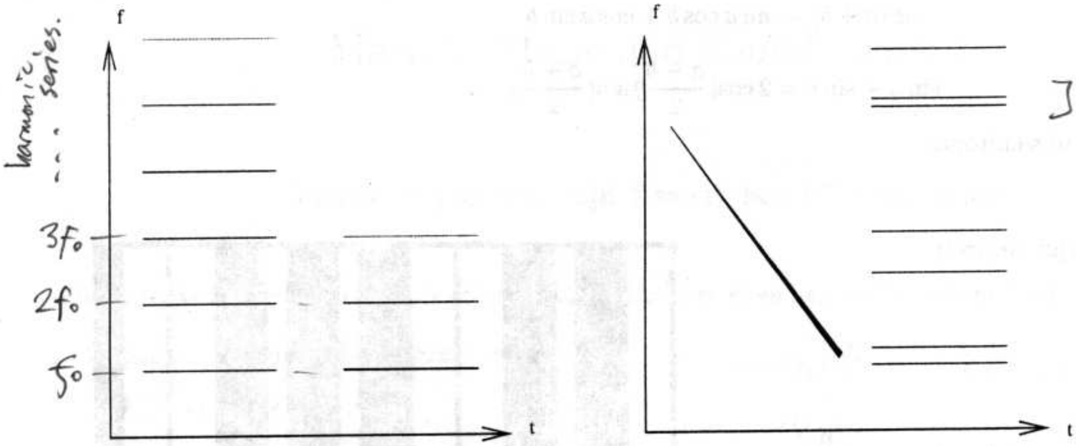
$$\sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2\cos\left(2\pi \frac{f_1-f_2}{2} t\right) \sin\left(2\pi \frac{f_1+f_2}{2} t\right)$$



multiply as functions

6

4. Describe in as much detail as you can what sounds these two spectrograms correspond to (discuss periodicity, pitch, timbre, etc)

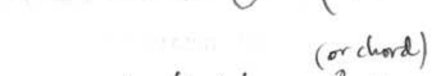


]– probably dissonant.

2 notes are sounding, with the same pitch but different timbre (harmonic content).

The pitch is clearly the same because the partials form a harmonic series in both cases, with the fundamental f_0 being the same.

Both are periodic signals since such signals have partials in a harmonic series. But their c_1, c_2, c_3 etc. coefficients are different.

A pure tone with frequency decreasing but amplitude increasing, for instance the graph could be  t.

This is followed by a bell-like sound (or chord), which is not a periodic signal (since the partials are not in a harmonic series). It may not have a well-defined musical pitch (there's no common divisor) It may be dissonant since there are partials near each other (within 10% in frequency).

The first will be harsh, the second more mellow.