

## Math 5: Music and Sound. Some final practise questions

You will be provided with the equations on the back sheet but no others

Please also use Midterm practise questions, quizzes, and HW to review.

1. What is the frequency of  $5 \sin(1000t + 2)$ ? What is the amplitude? What is its wavelength in air?
2. What is period and frequency of

$$g(t) = \begin{cases} 1, & \text{floor}(t) \text{ odd} \\ 0, & \text{floor}(t) \text{ even} \end{cases}$$

3. How many whole tones should fit in an octave using the chromatic scale? A Pythagorean whole tone is 9:8. Find the frequency error in cents in constructing an octave using this interval. How is this resolved in music practice?
4. Find  $B$  such that  $\sin \omega t + B \cos \omega t$  has a phase of  $\pi/3$ .
5. Compute the volume of a bottle with effective neck length 6 cm, neck area 2 cm<sup>2</sup>, and resonant frequency 100 Hz.
6. What ‘strike note’ will you perceive if a bell has partials 350, 557, 1109, 1666, 2500 Hz?
7. Compare the dissonance of the pair of periodic signals with an interval of a major 6th to a pair with an interval of a major 7th, using our usual rule (within 10% but not within 15 Hz). Take the lowest note in each pair to be 400 Hz, and assume partials are only relevant up to 2400 Hz.
8. Explain the following terms (to someone not in this course) giving any mathematical definitions if needed.
  - (a) diffraction
  - (b) Q factor
  - (c) reverberation time
  - (d) stretched partials
  - (e) formant
9. Draw the pressure graphs for the first 2 modes of a clarinet (uniform width open-closed pipe), and compare to those for a flute. What is the purpose of a register key? Where would you put one on a flute? (It doesn’t in fact use one). What if you wanted to jump to the next ( $n = 3$  mode) overtone for a flute—where would you put a register key then?
10. A sinusoid wave with frequency 100 Hz travels from left to right. Find the phase difference between two points separated by 1m. How far apart do you need to go so that the signals at the two points are in phase? (Give all such distances)
11. If I breathed a gas (say helium) that made the speed of sound twice as fast in my vocal tract, what would change when I spoke? (Discuss pitch of my voice and formants, sketch a spectrum before and afterwards)

12. A bell when struck produces a partial at 600 Hz which takes 1 sec to die by 10 dB in intensity. Compute the Q factor of the corresponding bell mode. What range of driving frequencies would cause significant (*i.e.* at least half the max) resonance of this mode?
13. The bell of a clarinet flares out at the bottom end. What do you expect this to do to the mode frequencies compared to a straight pipe?
14. If the vocal tract were widened down near the vocal cords, how would you expect formants to change? Would this apply the same to each formant?
15. The decay of a clap dies by 20 dB in 1.5 sec. Compute the reverb time. Will the hall sound dead, about right or echoey?
16. A room is a cuboid of dimensions 3 m by 4 m by 5 m. Compute the reverb time if the walls reflect 90% of the incident energy. Find the absorption coefficient that would give a reverb time of 2 sec.
17. If you stood in the middle of the above room and clapped, what would be the time that the first echo would return to you? Draw a diagram to show this.
18. A person stands 2 m from a reflective wall and speaks. A listener stands also 2m from the same wall, but 3m away from the speaker. Compute the two path lengths, and the time delay. Draw a diagram to show the image source. What frequencies in the spoken signal would interfere constructively? Destructively?
19. How far from a reflective wall would you need to stand so that if you sung a pure tone (not possible in reality) at 440 Hz, the reflected sound would destructively interfere with the direct sound? You can assume the direct sound has zero path length (your mouth and ears are at the same location).
20. what frequency range of sound would spread out in all directions traveling through an open doorway of width 0.6m? What angular width would a high frequency of 5000 Hz spread out to after propagating through this doorway?
21. If a C4 is played by a musician from a parade float moving at 20 m/s away from a listener, what note is perceived? Compute the cents error from the equal-tempered pitch.
22. Two such parade floats produce this same note, except one moves towards you at 5 m/s and the other moves away at 5 m/s. Assuming the notes produced are pure tones of equal amplitude, what would you expect to hear? (give any relevant new frequencies).
23. Draw a spacetime diagram illustrating why apparent frequency increases when you move towards a source of sound. What frequency is perceived if you move away at the speed of sound? What freq is perceived if a source of sound comes towards you at exact the speed of sound?
24. Why is ultrasound for medical imaging chosen to be 100 times higher in frequency than humans can hear?
25. Explain the difference between lightly touching a string and pressing hard onto the fretboard. If you lightly touch a string at 2/5 of the way down, what partials are in the resulting sound produced by the string, compared to the original string?
26. Compute the amplitudes of the first 5 modes when a string is plucked 2/5 of the way down. Which if any are absent?
27. [advanced] Explain why the oboe has all harmonics present yet a clarinet only has odd ones. Both behave as open-closed pipes.
28. An orchestra produces 5 W of total sound power. How many dB will you hear at a distance of 250 m? How far away would you need to go to increase the intensity by 10 dB? (use ratios). How much sound power is collected by your ear ( $5 \text{ cm}^2$ ) at a distance of 1000 m?
29. You clap in the middle of a square room of side 5m. Compute the first few arrival times of reflections from the wall. Don't forget multiple reflections. [advanced: write a formula for their full list].

## Useful information

$$\omega = 2\pi f$$

$$c = f\lambda$$

$$\text{dB} = 10 \log_{10} \frac{I}{10^{-12} \text{W/m}^2}$$

$$Q = \pi \frac{\tau}{T}$$

$$\frac{f_{\text{obs}}}{f} = \frac{1}{1 - v/c} \quad \text{or} \quad 1 + v/c$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

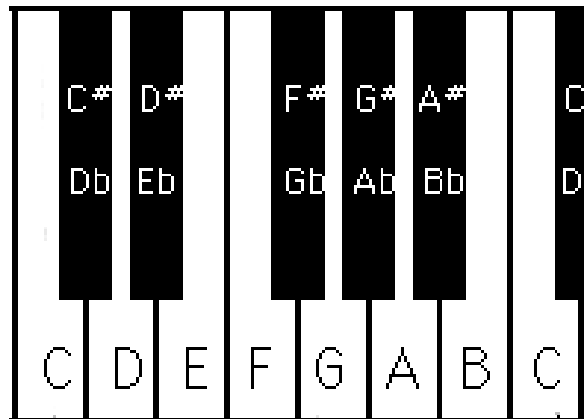
$$\sin a + \sin b = 2 \cos\left(\frac{a-b}{2}\right) \sin\left(\frac{a+b}{2}\right)$$

$$c_{\text{string}} = \sqrt{\frac{T}{\mu}}$$

$$f_{\text{Helm}} = \frac{c}{2\pi} \sqrt{\frac{a}{Vl}}$$

Intervals by number of semitones:

1. minor second
2. whole tone (major second)
3. minor third
4. major third
5. perfect fourth
6. tritone (augmented fourth)
7. perfect fifth
8. minor sixth
9. major sixth
10. minor seventh
11. major seventh
12. octave



The standard musical pitch A4 is 440 Hz

You can use the speed of sound as 340 m/s.