Math 56
X-Hour Review

Do the following questions:

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1. Decide whether computing $2x$ via the algorithm $x + x$ is backward stable.
2. Decide whether computing 1 via the algorithm $\frac{x}{x}$ is backward stable.
3. True or False? Prove your answer.
   (a) $\sin(x) = O(1)$ as $x \to \infty$
   (b) $\sin(x) = O(1)$ as $x \to 0$
   (c) $n^{\frac{1}{n}} = O(1)$ as $n \to \infty$
   (d) $\sin(1/n) = O(n^{-1})$ as $n \to \infty$
   (e) $(1+\epsilon)(1+\epsilon) = 1 + O(\epsilon)$ as $\epsilon \to 0$
4. Consider the linear system:
$$\frac{1}{2}\begin{pmatrix} 1001 & 999 \\ 999 & 1001 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$
   Suppose that we have a backward stable method to solve for the y vector (you may assume that the constant in the backward stable method is order 1). How many digits of accuracy (relative to $||y||$) do you expect in the solution using this backward stable method?
5. Consider the function:
$$\begin{cases} e^{\frac{-1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$
   It turns out this function is infinitely differentiable (so you can apply things like Taylor's Theorem).
   (a) Come up with a rigorous bound on the error when trying to use the first order Taylor series expansion at 0 to compute $f(1)$? Something you might find useful:
$$f'''(x) = \frac{4e^{-1/x^2}(6x^4 - 9x^2 + 2)}{x^9},$$
   for $x \neq 0$ and the only root of $6x^4 - 9x^2 + 2$ between 0 and 1 is roughly 0.52.
   (b) Can you come up with a guess for the Taylor series expansion of f centered at 0? Does the Taylor series for f centered at 0 converge to f at all?
   (c) What must be true about f so that we do not contradict another beloved theorem about convergence of Taylor series?