Math 56 Compu & Expt Math, Spring 2014: Quiz 2

in X-hr 5/7/14, 35 mins, just pencil and paper

1. (a) Compute the periodic convolution of $[1, 1, 2]$ with $[1, 0, -1]$.

    (b) To what length should these vectors be zero-padded so that their periodic convolution correctly computes the acyclic one?

2. Write down the middle row of the $N = 3$ DFT matrix (if you use symbols, define them).

3. (a) Say the function $f(x) = e^{-3ix}$ is sampled on a regular grid of size $N = 8$. What DFT coefficient vector $\tilde{f}$ would result?

(b) What function results when trigonometric polynomial interpolation on this same grid is used to reconstruct f from the DFT coefficients you just computed? Comment.

4. A function has Fourier series decaying as $\hat{f}_n = O(1/|n|^3)$. It is sampled on a regular N-point grid and the DFT taken. Prove an optimal big-O bound on the decay vs N of $|\tilde{f}_0 - \hat{f}_0|$, ie the error in the approximated average value.

[BONUS: prove a big-O bound on the interpolation error]

5. Let f and g be length-N signal vectors. What is $\tilde{f} * \tilde{g}$ in terms of f and g ? Prove it. [Hint: this is a kind of inverted convolution theorem.]