1. Prove Theorem M19.4: If X_α is Hausdorff for all $\alpha \in I$, then $\prod_{\alpha \in I} X_\alpha$ is Hausdorff when given either the box topology or the product topology.

2. Prove that the intervals $[0,1]$, $(0,1)$, and $[0,1)$ are not homeomorphic as subspaces of $\mathbb{R}$.

   Hint: If you remove the point $x = \frac{1}{2}$ from any of these sets, the resulting space is disconnected.

3. The goal of this problem is to show that differentiable functions whose derivatives vanish are locally constant.

   (a) Consider $(X, \mathscr{T}_d)$. Prove that the only connected sets are $\{x\}$ for $x \in X$.

   (b) Let X be connected and $f : X \to Y$ locally constant.[1] Prove that f is a constant function.

   (c) Let U be an open subset of $\mathbb{R}$ and $f : U \to \mathbb{R}$ a differentiable function such that $f' \equiv 0$ (i.e., $f'(x) = 0$ for all $x \in U$). Prove that f is locally constant.

   Hint/Warning: A function with vanishing derivative need not be constant.

4. (Chain Lemma) Assume $X = \bigcup_{n=1}^{\infty} X_n$ where each X_n is connected and $X_{n-1} \cap X_n \neq \varnothing$ for all $n \in \mathbb{Z}_+$. Prove that X is connected.

5. Let $f : X \to Y$ be a continuous function. Prove that if X is path connected then $f(X)$ is path connected.

6. (Brouwer Fixed-Point Theorem)[2] Prove that any continuous function $f : [-1,1] \to [-1,1]$ has a fixed point. That is, $\exists x$ so that $f(x) = x$.

[1] We defined locally constant functions in Homework 7.

[2] This is the 1-dimensional version of this theorem. We will hopefully cover the more general version later in class.