1. (Subspace open and closed sets.) Consider $\mathbb{R}$ (with its usual topology). Let $A = [0, 6]$ and $B = [0, 2) \cup \{3, 4\}$ be subspaces of $\mathbb{R}$. Determine whether the following sets are open, closed, clopen, or neither in each subspace (justify by showing what open/closed set works):

	$[0,1)$	$\{3,4\}$	$[1,2)$
A			
B			

2. For each set listed, find the interior, boundary, and closure in each of the listed spaces (no justification is *required*!):

- $A = [0, 1) \cup (1, 2)$.

	$\operatorname{Int} A$	$\operatorname{Bd} A$	$\overline{A}$
$\mathbb{R}$			
$\mathbb{R}_\ell$			
$(\mathbb{R}, \mathscr{T}_d)$			

- $A = [0, 1) \times (0, 1)$.

	$\operatorname{Int} A$	$\operatorname{Bd} A$	$\overline{A}$
$\mathbb{R} \times \mathbb{R}$			
$\mathbb{R}_\ell \times \mathbb{R}$			

Hint: Draw pictures of the set A and what the typical open sets in each space look like.

3. Consider $(\mathbb{R}, \mathscr{T}_f)$. Let $A \subset \mathbb{R}$ be infinite. Show that every point $x \in \mathbb{R}$ is a limit point of A.

4. Consider $\mathbb{R}$ (with its usual topology). By using the interior and closure operations, we can obtain different sets. What happens when we use these operators repeatedly?

   (a) Find a set $A \subset \mathbb{R}$ so that $A, \operatorname{Cl} A,$ and $\operatorname{Int} A$ are pairwise distinct.

   (b) Find a set $A \subset \mathbb{R}$ so that we obtain 4 pairwise distinct sets by applying combinations of Int and Cl to A (e.g., A, $\operatorname{Cl} A$, $\operatorname{Int} A$, and $\operatorname{Cl} \operatorname{Int} A$).

   (c) Find a set $A \subset \mathbb{R}$ so that we obtain 5 pairwise distinct sets in this way.

   (d) (Optional/Bonus) Determine the maximum number of pairwise distinct sets that can be obtained in this way and prove it. Along the way, share an example of a set that obtains this maximum.