1. **Let $(X, \mathscr{T})$ be a topological space and $A \subset B \subset X$. There are two topologies we could naturally place on A:**

   (1) The subspace topology $\mathscr{T}_A$ coming from $\mathscr{T}$,

   (2) The subspace topology $(\mathscr{T}_B)_A$ coming from $(B, \mathscr{T}_B)$.

   Show that these two topologies are the same.

2. **Define $N_{a,b} = \{an + b \mid n \in \mathbb{Z}\}$ (so $N_{a,b} \subset \mathbb{Z}$) and let $\mathscr{N} = \{N_{a,b} \mid \gcd(a, b) = 1\}$. Prove that:**

   (a) $\mathscr{N}$ is a basis for a topology on $\mathbb{Z}$.

   (b) Every open set in this topology (other than $\varnothing$) has infinitely many elements.

3. **Consider $(X, \mathscr{T}_d)$ and $(Y, \mathscr{T}_t)$. Describe the product topology on $X \times Y$ without mention of a basis.**

4. **Let $(X_i, \mathscr{T}_i)$ be a topological space for $i = 1, 2, 3$. Define a product topology on $X_1 \times X_2 \times X_3$. Then generalize this definition.**

5. **Determine whether each of the following sets is open in each of $\mathbb{R}, \mathbb{R}_\ell, (\mathbb{R}, \mathscr{T}_d), (\mathbb{R}, \mathscr{T}_f)$, and $(\mathbb{R}, \mathscr{T}_t)$:**

   - $A = \{x \in \mathbb{R} \mid x \neq \pi, -\pi\}$.
   - $B = \{x \in \mathbb{R} \mid x \notin \mathbb{Z}\}$.