1. **Let** $\mathscr{C} = \{(a,b) \mid a,b \in \mathbb{Q}\}$**. This collection is a basis for a topology on** $\mathbb{R}$**. Prove that:**

   (a) $\mathscr{T}_{\mathscr{C}}$ **is the usual topology on** $\mathbb{R}$**.**

   (b) $\mathbb{R}$ **has a countable basis (i.e., a basis whose cardinality is countable).**

2. **Let** $\mathscr{D} = \{[a,b) \mid a,b \in \mathbb{Q}\}$**. Prove that:**

   (a) $\mathscr{D}$ **is a basis for a topology on** $\mathbb{R}$**.**

   (b) $\mathscr{T}_{\mathscr{D}}$ **is different from the lower limit topology (i.e., the topology on** $\mathbb{R}_\ell$**).**

3. **Define a relation "odd is always greater" on** $\mathbb{Z}$ **by:**

$$a \prec b \Leftrightarrow \begin{cases} a < b \text{ and both } a \text{ and } b \text{ are even} \\ a < b \text{ and both } a \text{ and } b \text{ are odd} \\ a \text{ is even and } b \text{ is odd} \end{cases}.$$

   (a) **Prove that** $\prec$ **is a linear order on** $\mathbb{Z}$**.**

   (b) **Explicitly describe the following sets:**

   - $(1,9) =$
   - $(0,1) =$
   - $(-\infty,0) =$

   (c) **Prove that** $\mathscr{T}_{\prec} = \mathscr{T}_d$**.**