1. **Let A and B be sets. Prove or disprove each of the following statements:**

   (a) $\mathscr{P}(A \cup B) = \mathscr{P}(A) \cup \mathscr{P}(B)$**,**

   (b) $\mathscr{P}(A \cap B) = \mathscr{P}(A) \cap \mathscr{P}(B)$**,**

   (c) $\mathscr{P}(A \setminus B) = \mathscr{P}(A) \setminus \mathscr{P}(B)$**.**

   Solution or

   Proof. Proof. □

2. **Let $f : X \to Y$ be a function. Prove or disprove each of the following:**

   (a) If $B \subset Y$ **then** $f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$**.**

   (b) For any collection $\{A_\alpha \mid A_\alpha \subset X\}_{\alpha \in I}$**,**

   $$f(\bigcap_{\alpha \in I} A_\alpha) = \bigcap_{\alpha \in I} f(A_\alpha).$$

   (c) For any collection $\{B_\beta \mid B_\beta \subset Y\}_{\beta \in J}$**,**

   $$f^{-1}(\bigcap_{\beta \in J} B_\beta) = \bigcap_{\beta \in J} f^{-1}(B_\beta).$$

3. **Prove that $\mathbb{Q}$ is countably infinite.**

4. **Find a bijection between the intervals $[0, 1]$ and $(0, 1)$.**

5. **Let $(X, \mathscr{T})$ be a topological space. Prove that a set A is open if and only if**

   for all $p \in A$ there is an open set $U_p \in \mathscr{T}$ such that $p \in U_p \subset A$.

6. **Let $X = \{a, b, c, d\}$ and $\mathscr{T} = \{\varnothing, \{a\}, \{b, c\}, \{c, d\}, \{a, b, c\}\}$. Explain why $(X, \mathscr{T})$ is not a topological space and then modify $\mathscr{T}$ to make it a topology on X.**