MATH 54 - TOPOLOGY
SUMMER 2015
MIDTERM 1

DURATION: 1 HOUR 30 MINUTES

This exam consists of 4 independent problems. Treat them in the order of your choosing, starting each problem on a new page.

Every claim you make must be **fully justified or quoted** as a result from the textbook.

Problem 1

Let $\mathcal{T}$ be the family of subsets $\mathcal{U}$ of $\mathbb{Z}_+$ satisfying the following property:

If n is in $\mathcal{U}$, then any divisor of n belongs to $\mathcal{U}$.

1. Give two different examples of elements of $\mathcal{T}$ containing 24 (not including $\mathbb{Z}_+$).

2. Verify that $\mathcal{T}$ is a topology on $\mathbb{Z}_+$.

3. Is $\mathcal{T}$ the discrete topology?

Problem 2

Let (E, d) be a metric space.

1. Recall the definition of the metric topology and prove that open balls form a basis.

2. Assume that ρ is a second metric on E such that, for every $x, y \in E$,

$$\frac{1}{2}d(x,y) \leq \rho(x,y) \leq 2d(x,y).$$

Compare the topologies generated by d and ρ.

Problem 3

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be topologies on a set X.

1. Verify that $\mathcal{T}_1 \cup \mathcal{T}_2$ is a subbasis for a topology.

From now on, $\mathcal{T}_1 \vee \mathcal{T}_2$ denotes the topology generated by $\mathcal{T}_1 \cup \mathcal{T}_2$.

2. Describe $\mathcal{T}_1 \vee \mathcal{T}_2$ when $\mathcal{T}_1$ is coarser than $\mathcal{T}_2$.

3. Compare $\mathcal{T}_1 \vee \mathcal{T}_2$ with $\mathcal{T}_1$ and $\mathcal{T}_2$ in general.

4. Let $\mathcal{T}$ be a topology on X that is finer than $\mathcal{T}_1$ and $\mathcal{T}_2$.
Prove that $\mathcal{T}$ is finer than $\mathcal{T}_1 \vee \mathcal{T}_2$.

Problem 4

1. Consider the set $Y = [-1, 1]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y? Which are open in $\mathbb{R}$?

$$A = \left\{ x \ , \ \frac{1}{2} < |x| < 1 \right\}$$

$$B = \left\{ x \ , \ \frac{1}{2} < |x| \leq 1 \right\}$$

$$C = \left\{ x \ , \ \frac{1}{2} \leq |x| < 1 \right\}$$

$$D = \left\{ x \ , \ 0 < |x| < 1 \text{ and } \frac{1}{x} \in \mathbb{Z}_+ \right\}$$

2. Let $X = \mathbb{R}_\ell \times \mathbb{R}_u$ where $\mathbb{R}_\ell$ denotes the topology with basis consisting of all intervals of the form $[a, b)$ and $\mathbb{R}_u$ denotes the topology with basis consisting of all intervals of the form $(c, d]$.
Describe the topology induced on the plane curve Γ with equation $y = e^x$.