Math 54 Summer 2015
Homework #4: closed sets and limit points

(1) Prove the following result:
Theorem Let X be a set and $\gamma : \mathcal{P}(X) \to \mathcal{P}(X)$ a map such that, for any $A, B \subset X$,
(i) $\gamma(\emptyset) = \emptyset$;
(ii) $A \subset \gamma(A)$;
(iii) $\gamma(\gamma(A)) = \gamma(A)$;
(iv) $\gamma(A \cup B) = \gamma(A) \cup \gamma(B)$.
Then the family $\{X \setminus \gamma(A)\,,\, A \in \mathcal{P}(X)\}$ is a topology in which $\overline{A} = \gamma(A)$.
Hint: it might be useful to prove that $A \subset B \Rightarrow \gamma(A) \subset \gamma(B)$.

(2) *The questions in this problem are independent.*
(a) Show that a topological space X is Hausdorff if and only if the diagonal $\Delta = \{(x,x)\ ,\ x \in X\}$ is closed in $X \times X$.
(b) Determine the accumulation points of the subset $\left\{\frac{1}{m} + \frac{1}{n}\ ,\ m, n \in \mathbb{Z}_+\right\}$ of $\mathbb{R}$.

(3) Treat Problem **17.19** in the textbook.

(4) Treat Problem **17.20** in the textbook.