Math 54 Summer 2015
Homework #3: topological spaces

(1) Let $\{\mathcal{T}_\alpha\}_{\alpha \in A}$ be a family of topologies on a non-empty set X.

a. Prove that $\mathcal{I} = \bigcap_{\alpha \in A} \mathcal{T}_\alpha$ is a topology on X.

b. Prove that $\mathcal{I}$ is the finest topology that is coarser than each $\mathcal{T}_\alpha$.

(2) Let p be a prime number. Consider for $n \in \mathbb{Z}$ and a a positive integer,

$$B_a(n) = \{n + \lambda p^a \ , \ \lambda \in \mathbb{Z}\}.$$

a. Show that the family $\mathcal{B} = \{B_a(n) \ , \ n \in \mathbb{Z} \ , \ a \in \mathbb{Z}_+\}$ is a basis for a topology.

b. Is the topology generated by $\mathcal{B}$ discrete?

(3) Treat Problem **13.7** in the textbook.

(4) Treat Problem **16.8** in the textbook.
Note: the answer depends on the slope of the line.