

Consider  $\vec{f}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/2 \\ 2y - 7x^2 \end{pmatrix}$  map in  $\mathbb{R}^2$

A) find formula for  $\vec{f}^{-1}\begin{pmatrix} x \\ y \end{pmatrix}$

B) Sketch the set  $S = \{(x, 4x^2) : x \in \mathbb{R}\}$

Show  $S$  is invariant under  $\vec{f}$

ie  $\vec{x} \in S \Rightarrow \vec{f}(\vec{x}), \vec{f}^{-1}(\vec{x}) \in S$ .

C) Show  $S$  is either stable or unstable manifold — which? (where's fixed pt?)

D) What is the other manifold? (Hint: fix  $x=0$ )

E) Show no points outside  $S$  converge to  $\vec{0}$  under  $\vec{f}$  or  $\vec{f}^{-1}$

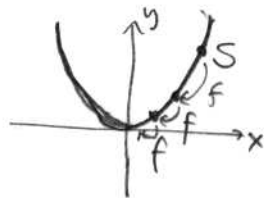
SOLUTIONS

Consider

$$\vec{f} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/2 \\ 2y - 7x^2 \end{pmatrix}$$

map in  $\mathbb{R}^2$

A) find formula for  $\vec{f}^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y/2 + 14x^2 \end{pmatrix}$



Define

$$\vec{f} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ i.e.}$$

$$\text{solve for } u, v \begin{cases} u/2 = x \\ 2v - 7u^2 = y \end{cases}$$

$$u = 2x$$

$$v = \frac{y + 7u^2}{2} = \frac{y + 28x^2}{2} = \frac{y}{2} + 14x^2$$

B) Sketch the set  $S = \{(x, 4x^2) : x \in \mathbb{R}\}$

Show  $S$  is invariant under  $\vec{f}$

$$\text{i.e. } \vec{x} \in S \Rightarrow \vec{f}(\vec{x}), \vec{f}^{-1}(\vec{x}) \in S.$$

$$\vec{f} \begin{pmatrix} x \\ 4x^2 \end{pmatrix} = \begin{pmatrix} x/2 \\ 2(4x^2) - 7x^2 \end{pmatrix}$$

$$= \begin{pmatrix} x/2 \\ x^2 \end{pmatrix} \in S \text{ since applying "4x^2" to x/2 gives x^2.}$$

$$\vec{f}^{-1} \begin{pmatrix} x \\ 4x^2 \end{pmatrix} = \begin{pmatrix} 2x \\ \frac{4x^2}{2} + 14x^2 \end{pmatrix} = \begin{pmatrix} 2x \\ 16x^2 \end{pmatrix} \in S.$$

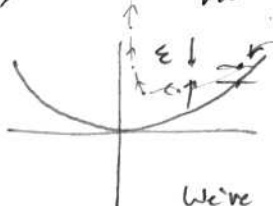
C) Show  $S$  is either stable or unstable manifold — which? (where's fixed pt?)  
fixed point is  $\vec{0}$ . since  $\begin{pmatrix} x \\ 4x^2 \end{pmatrix} \rightarrow \begin{pmatrix} x/2 \\ x^2 \end{pmatrix} \rightarrow \begin{pmatrix} x/4 \\ x^2/4 \end{pmatrix} \dots$  has limit  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

D) What is the other manifold? (Hint: fix  $x=0$ )

$$U(\vec{0}) = y\text{-axis. } \{ \begin{pmatrix} 0 \\ y \end{pmatrix} : y \in \mathbb{R} \} \text{ since } \vec{f}^{-n} \begin{pmatrix} 0 \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ in limit.}$$



E) Show no points outside  $S$  converge to  $\vec{0}$  under  $\vec{f}$  or  $\vec{f}^{-1}$



Any point can be written

$$\begin{pmatrix} x \\ 4x^2 + \epsilon \end{pmatrix} \xrightarrow{f} \begin{pmatrix} x/2 \\ x^2 + 2\epsilon \end{pmatrix} \rightarrow \begin{pmatrix} x/4 \\ x^2/4 + \epsilon \end{pmatrix} \dots \rightarrow \begin{pmatrix} x/2^n \\ x^{2^n}/4^n + \epsilon \end{pmatrix} + \begin{pmatrix} 0 \\ 2^n \epsilon \end{pmatrix}$$

We've done a nonlinear coordinate change  $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ \epsilon \end{pmatrix}$  that makes map linear with  $A = \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}$ !

blows up unless  $\epsilon = 0$ .