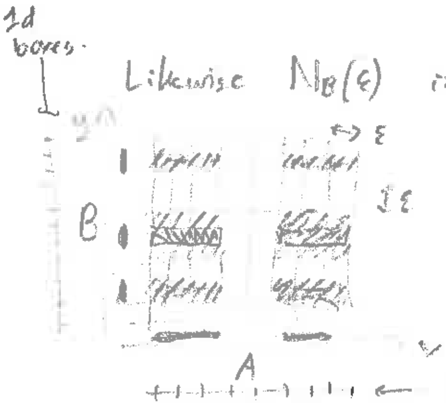
HW6, some solutions.

11/4/15
Barnett

4.10) Defn. of $\text{boxdim}(A) = \lim_{\varepsilon\to 0} \frac{\ln N_A(\varepsilon)}{\ln 1/\varepsilon}$

where $N_A(\varepsilon)$ is, at any ε, the # one-dimensional boxes touching the set A

Likewise $N_B(\varepsilon)$ is # 1d boxes covering B



$N_{A\times B}(\varepsilon) = N_A(\varepsilon)\, N_B(\varepsilon)$

boxes touching $A\times B$ in 2d.

since if $x\in X$ is in a 1d box in x, and $y\in Y$ is in 1d box in y, $(x,y)\in X\times Y$ is in the 2d box, and this is the only way this 2d box can be touched.

To define $\text{boxdim}(A\times B)$, we need to choose fixed shape of boxes as $\varepsilon\to 0$. Easiest way is pick ε same in x & y, i.e. square boxes. (some of you had $\varepsilon_A, \varepsilon_B \to 0$ independently, not good)

$$\text{boxdim}(A\times B) = \lim_{\varepsilon\to 0}\frac{\ln N_{A\times B}(\varepsilon)}{\ln 1/\varepsilon} = \lim_{\varepsilon\to 0}\frac{\ln(N_A(\varepsilon)\,N_B(\varepsilon))}{\ln 1/\varepsilon}$$

$$= \lim_{\varepsilon\to 0}\frac{\ln N_A(\varepsilon)}{\ln 1/\varepsilon} + \lim_{\varepsilon\to 0}\frac{\ln N_B(\varepsilon)}{\ln 1/\varepsilon}$$

using properties of log & of limits.

$$= \text{boxdim}(A) + \text{boxdim}(B).$$