10/31/09
Barnett

MATH 53 HW 5 Solutions

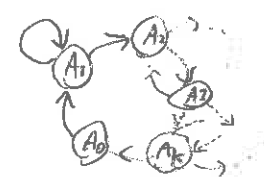
Chal 3, Step 4 (as reworded in HW5)

i) Naively you know $f^k(A_1) = I \supset A_1$ so $\exists$ f.p. of f^k in A_1
$\uparrow$ by defn of k

but we need to exclude the f.p. of f that we know already exists in A_1.

We do this by leaving A_1 at some point, eg. by going to A_0 (the interval from Step 3 that maps back to A_1, ie $f(A_0) \supset A_1$)

So if $k = p-3$, $f^{p-3}(A_1) \supset A_0$ and $f(A_0) \supset A_1$



so $f^{p-2}(A_1 \underbrace{S \cdots S}_{p-4 \text{ steps}} A_0 A_1) \supset A_1$

One proof this cannot correspond to a lower period (divisor of $p-2$) is that in order to make it to A_0 in k steps it must enter a new interval A_n each step.
$\Rightarrow \exists$ period $(p-2)$ in A_1 by Cor. 3.18. (so $S \cdots S = A_2 A_3 \cdots A_k$)

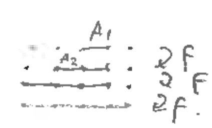
Similarly, if $k < p-3$ you are able to add $p-3-k$ extra A_1's at start to make a chain of the same length $p-2$: $f^{p-2}(\underbrace{A_1 A_1 \cdots A_1}_{p-2-k} \underbrace{A_2 \cdots A_k}_{k-1} A_0 A_1) \supset A_1$
$\Rightarrow \exists$ period $(p-2)$ in A_1

ii) $f^n(A_2) \supset A_1$ means there's a route from A_2 back to A_1 in n steps

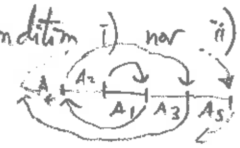
$f^{p-2}(\underbrace{A_1 A_1 \cdots A_1}_{p-2-n} \underbrace{A_2 A_3 \cdots A_{n+1}}_{n} A_1) \supset A_1$ $\Rightarrow \exists$ period $p-2$
(since $A_2 \cdots A_{n+1}$ must be a new interval each step, can't factor into a divisor, as above.)

in my question I meant there is some $n < p-2$, not for all $n < p-2$.

If confused, the best way to work all this out was with $p = 5$ example:



BONUS: if period $(p-2)$ does not exist, neither condition i) nor ii) can hold, so $k = p-2$ & the smallest n s.t. $f^n(A_2) \supset A_1$ is $p-1$. So,



spirals is only way to have this.

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10/28/11

Solution to A) in HW5. (Math 53)

Use $|a+b| \geqslant \big| |a| - |b| \big|$ follows from tri. ineq.

$z_{n+1} = z_n^2 + c$

so $|z_{n+1}| \geqslant \underbrace{|z_n^2|}_{= |z_n|^2} - |c|$

$\Rightarrow \dfrac{|z_{n+1}|}{|z_n|} \geqslant |z_n| - \dfrac{|c|}{|z_n|}$ since $|c| < 2$, $|z_n| > 2$, this term < 1, so...

$> |z_n| - 1 \quad (*)$

> 1 since $|z_n| > 2$.

But note merely having each ratio strictly greater than 1 is not enough to prove $|z_n| \to \infty$. As an example $z_n = 1 - 2^{-n}$ remains bounded!

So, need more strength. Since $|z_n| > 2$, $\exists \varepsilon > 0$ s.t. $|z_n| > 2 + \varepsilon$

Returning to $(*)$, using this:

$$\frac{|z_{n+1}|}{|z_n|} > 2 + \varepsilon - 1 = 1 + \varepsilon$$

By induction, $\dfrac{|z_{n+m}|}{|z_n|} > (1+\varepsilon)^m$ so as $m \to \infty$, this tends to ∞, so $|z_n| \to \infty$ as $n \to \infty$.