Math 53 Chaos!: Homework 6

due Thurs Oct 29 . . . but best if do relevant questions after each lecture

Lots of Matlab, but much uses old or given chunks of code, and will help your projects. In last qu we go to continuous-time; Sec. 7.1-7.2 in the book are review of material from Math 23, so please read this.

Compu Expt 4.3: Write a code which plots the Julia set on on a pixel grid, for the c value from last HW, on the domain $|\text{Re } z_0| < 1.5$, $|\text{Im } z_0| < 1.5$, with some reasonable resolution, such as 0.01, using at least 200 iterations. Print out the Julia set and your code. BONUS: How many arms do the spirals have? How many 'spikes' seem to come together where spikes meet? You may need to zoom in to check these. [Hint: you will find iterating all points in a grid *at once* more efficient than what's suggested, although any method is fine. Given 1D list of grid values `x`, I recommend you use `[xx,yy] = meshgrid(x,x); zz = xx + 1i*yy;` to construct a 2D grid `zz` of complex numbers, which may be iterated *simultaneously*, similarly to if you had one number. Also check the `isnan` command, and `imagesc(x,x,...)` to plot. BONUS: improve efficiency by avoiding the `NaN`s which slow it down.]

T4.9 (see Example 4.5)

T4.11 b only (please make your proof hold water)

4.7 For a, naively there are two ways to create your ϵ and $N(\epsilon)$ sequence: one is wrong—think *very* carefully about which one is wrong by going as deep as K_5 and asking if *all* your $N(\epsilon)$ are needed to cover K_∞. For b, careful: carpet not gasket.

4.9 (easy)

4.10 [Hint: think about T4.9]

4.12 (isn't this bizarre? Part a is certainly not a fractal)

A. Numerically estimating box-counting dimension of chaotic Hénon attractor. I have given you almost all of the code to do this, so it should not be a hard matlab exercise. Download the code `henon_boxdim_hw6.m` and fill in the first part to fill N iterates of the Hénon attractor with $a = 1.4$ and $b = 0.3$. The second part of the code (which relies on the code `boxcount.m`) estimates the boxdim. You will notice a 'plateau' region in the plot of the slope $\log N(\epsilon)/\log(1/\epsilon)$. You will need to choose N large (eg start with 10^3 and go up) so that the attractor 'fills in' properly - how large an N did you need so that the plateau region stabilizes? Print out your plot.

[BONUS: generate either a different fractal as a set of points `x`, or use the examples on the boxcount webpage to grab a natural fractal image (choose as high resolution as possible), and estimate its boxdim. Eg coastline, tree, lightning strike. . .]

B. Numerically estimating correlation dimension of chaotic Hénon attractor. Generate an orbit of length $N = 10000$ (using the above parameters). Now compute for $r = 0.1$ and $r = 0.03$ the values of $C(r)$. Use this to estimate the correlation dimension. Is is close to that claimed in the text? Hint: you only need to write one simple loop, if I give you a command which returns the number of points in the list `x` which are within distance `r` of the n^{th} point `x(:,n)`. That command is

```
numel(find(sum((kron(x(:,n), ones(1,N)) - x).^2, 1) < r^2))
```

It will only work if `x` has exactly the size 2-by-N. Your code should take about 30 seconds to run, so please debug if it takes much longer.

T7.1 (review)