Math 53 Chaos! Fall 2015: Homework 1

due Thurs Sep 24 ... but best if do relevant questions after each lecture

Please collaborate on ideas, but write up individually. If still stuck, come to office hours or email me. Problems are from Alligood-Sauer-Yorke unless labelled A, B, etc. Remember to show your working/reasoning—answers without explanation will not receive a high score!

Some of your time this week is devoted to getting started with Matlab, a versatile and powerful package which I promise will bring you joy and success in your future careers! For help, always start with our course website http://math.dartmouth.edu/~m53f11/res.html, then ask friends or myself.

A. Install Matlab on your personal machine, *e.g.* from http://caligari.dartmouth.edu/downloads/matlab

Susan Schwarz can help with installation. Instead you could work at computer labs where Matlab is already installed. Make sure you follow the commands in the file `intro53.m`, and look through other introductions linked on our course site.

T1.3

T1.4 [remember: geometric]

T1.5 [Hint: use all existing info on fixed points of f]

B. [see Computer Expt 1.2] Download the incomplete Matlab code `iter_hw1.m` and finish it so that it iterates the map $f(x) = ax(1-x)$ with $a = 3.5$. I already included code to produce cobweb and iteration plots. Hand in printouts of these two plots for this map, for the initial condition $x_0 = 0.7$.

a) Try several different initial conditions and draw some conclusion about the basin of the observed orbit (what is its period?)

b) Try initial condition $x_0 = (a-1)/a = 0.714285714285714\ldots$ and explain what's happening!

T1.8 [Peek at back for a hint, but then you have to explain exactly where each term comes from].

T1.11 This is a key question. You can stop at $k = 4$.

1.1

1.2

1.4

1.9 [Hint: sketch the function]

1.15 It is rather mindblowing that this chaotic map has such a formula! We'll explore more later. [Hint: use the answer to guess a formula relating an angle θ_n to x_n. Find the absurdly simple update rule for θ_n. The 1.15 answer in the book is to the wrong question].

p.31 : Computer Expt 1.4: use the code above to investigate sensitive dependence as asked for. At a baseline you can keep rerunning this code and comparing by eye. You need only report results for separations 0.001 and 10^{-15}. BONUS: improve the code to compare *two* runs (using two colors) with separation 10^{-15} on the *same* iteration graph, choose nice axis range, and print out.