Worksheet #8: Binary numbers

(1) Find 61 in binary.

$2^5 + (61-32) = 2^5 + 29 = 2^5 + 2^4 + (29-16) = 2^5 + 2^4 + 13$

$= 2^5 + 2^4 + 2^3 + 5 = 2^5 + 2^4 + 2^3 + 2^2 + 0 \cdot 2^1 + 2^0$

$\Rightarrow$ 61 is 111101 in binary.

(2) What is the algorithm for computing the binary form of a general number (e.g. 17513)?

find the largest power k st 2^k divides into # st

$= 2^k$ + remainder. Proceed with remainder.

read off coefficients of 2^l for $l = k, k+1, \ldots, 0$.

(3) What fraction is the binary number $0.\overline{101}$?

real $x = 0.101101101$ ← binary

$2^3 x = 101.101101$ convert to real $5 + x$

$2^2 2^1 2^0$

$\Rightarrow 8x = 5 + x \Rightarrow 7x = 5 \Rightarrow x = 5/7$

(4) Find the binary expansion of $\frac{1}{7}$. start over

multiply by 2 until you get something > 1

$\frac{1}{7} \to \frac{2}{7} \to \frac{4}{7} \to \frac{8}{7} \to \frac{1}{7} \to \frac{2}{7} \to \frac{4}{7} \to \frac{8}{7} \to \frac{1}{7}$

$1 \cdot 2^{-3}$ $\quad 1 \cdot 2^{-6}$

$\Rightarrow$ in binary $\frac{1}{7}$ is $0.\overline{001}$

(5) Show how the algorithm for getting the binary expansion is the same as applying the $2x \pmod 1$ map.

Problem 4 explains this.

(6) So precisely which $x \in [0, 1]$ give chaotic orbits?

Chaotic orbits are given by irrational #'s.