Worksheet #6: Stable and unstable manifolds

Let $\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} x/2 \\ 2y - 7x^2 \end{bmatrix}$.

(1) Find an equation for $\boldsymbol{f}^{-1}(\boldsymbol{x})$.

(2) Sketch a graph of $S = \{(x, 4x^2) : x \in \mathbb{R}\}$. Show that S is invariant under $\boldsymbol{f}$ (i.e., $\boldsymbol{x} \in S$ implies $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{f}^{-1}(\boldsymbol{x})$ are in S.

(3) Is S a stable or unstable manifold? Show why this is the case.

(4) What is the other manifold? (Hint: fix $x = 0$)

(5) Show that no points outside of S converge to $\boldsymbol{0}$ under $\boldsymbol{f}$ or $\boldsymbol{f}^{-1}$.