

Worksheet #5: 2D linear stability

(1) Consider $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Find a condition on the eigenvalues of A such that $\mathbf{p} = \mathbf{0}$ is a
(a) sink:

(b) source:

(c) saddle point

(2) For $A = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$ write down and plot the first two iterates of $\mathbf{x}_0 = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$. What curve do they lie on?

(3) For $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$, verify that $A^n = a^{n-1} \begin{bmatrix} a & n \\ 0 & a \end{bmatrix}$.

(4) Write out $A^n \mathbf{x}$. Use this to decide a condition on a such that the fixed points are a sink or a source.