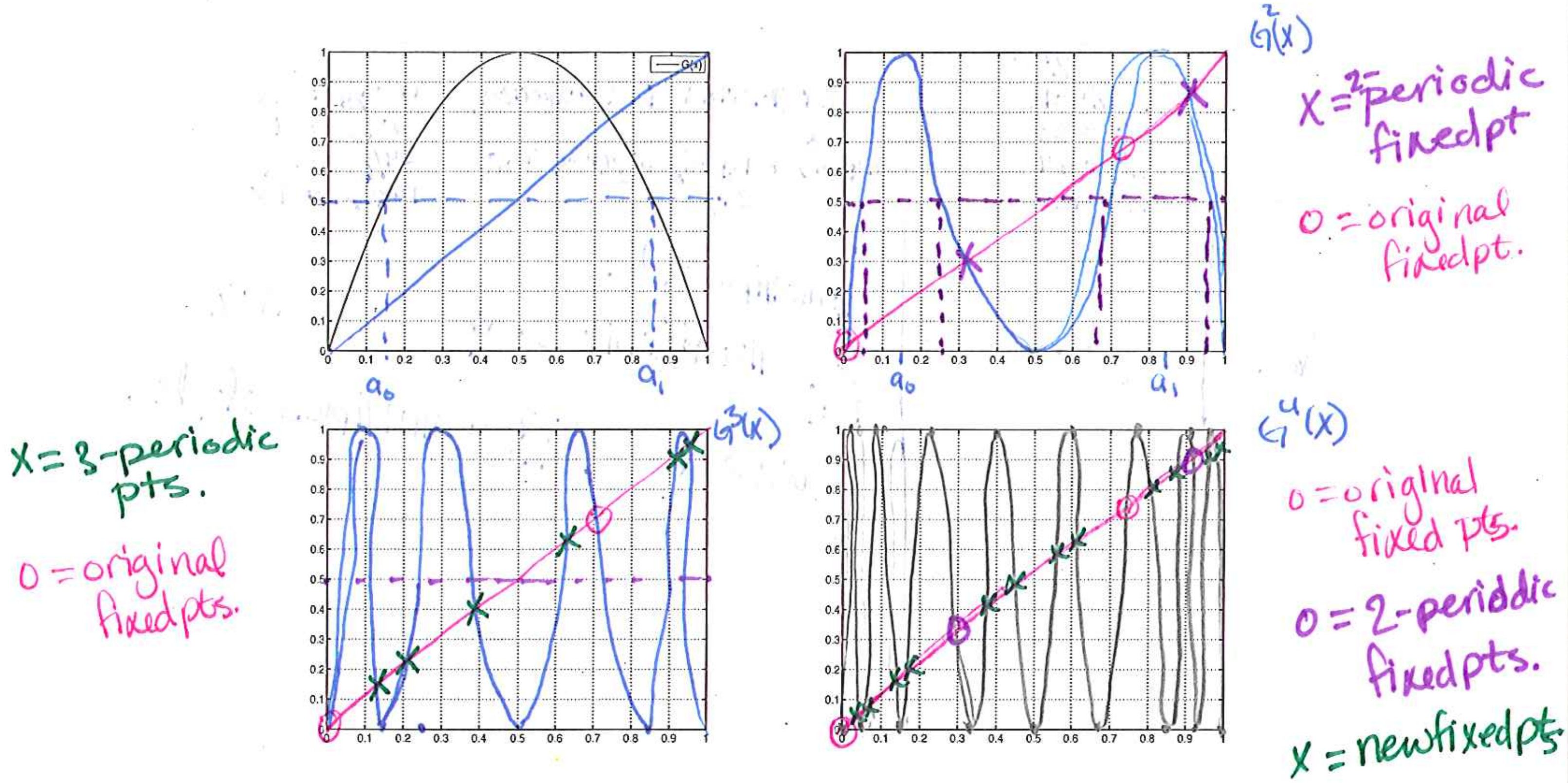
Worksheet #3: Counting periodic orbits

Consider the logistic function $G(x) = 4x(1-x)$. Plot $G^2(x)$, $G^3(x)$ and $G^4(x)$.



(1) How many fixed points of G^3 should there be?

$2^3 = 8$

(2) How many fixed points of G^3 are in 1-period orbits? 2-period orbits? [Hint: which lower periods give fixed points of G^3?]

2 - are fixedpts of $G(x)$.
0 - are fixedpts of $G^2(x)$

(3) So how many period 3 orbits are there?

$\frac{8-2}{3} = 2$ - 3-period orbits.

(4) How many fixed points of G^k are there?

2^k fixedpts. ← you get to prove this in the homework.

Complete the periodic table.

period-k	# of fixed points of G^k	# of fixed pts due to lower periods	# of k -periodic orbits
1	2	0	2
2	4	2	1
3	$2^3 = 8$	2	2
4	$2^4 = 16$	2(#P-1) + 2(#P-2) = 4	$16 - 4/4 = 3$
5	$2^5 = 32$	2	$30/5 = 6$
6	$2^6 = 64$	2(#P1) + 2(#P2) + 3·(#P3) = 10	$54/6 = 9$
7	$2^7 = 128$	2	$126/7 = 18$

k | 2^k | $\sum_{i=1}^{\#\text{multiples of } k}$ # of γ_i orbits $\cdot \gamma_i$

where $\{\gamma_i\}$ = multiples of k.