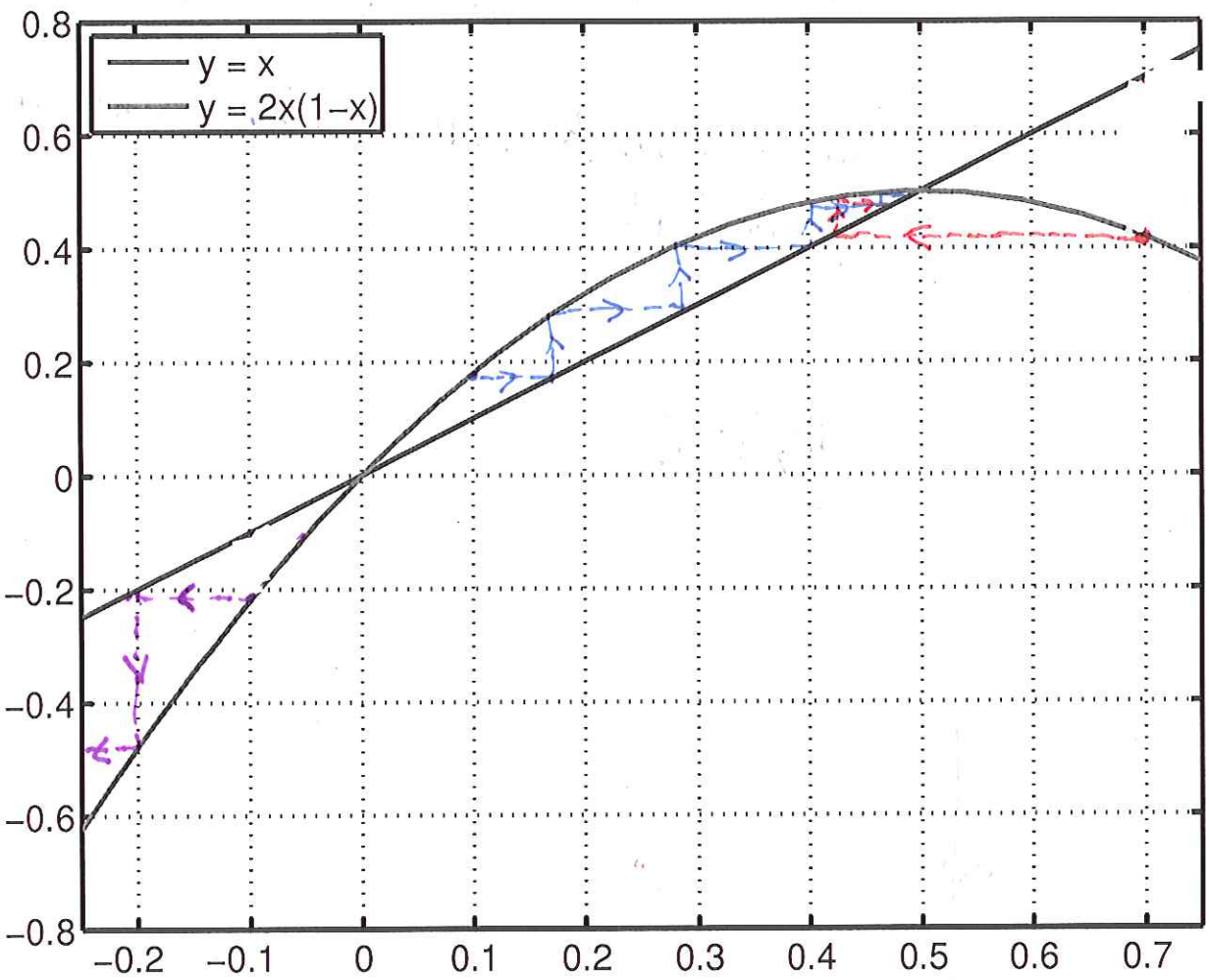
Worksheet #2: Scaling

(1) Let $f(x) = 2x(1-x)$. Find $f^2(x)$ (simplify to a polynomial), and $f^3(x)$ (you don't have to simplify this one).

$f^2(x) = f(f(x)) = -8x^4 + 16x^3 - 12x^2 + 4x$

$f^3(x) = f^2(f(x)) = -8(2x(1-x))^4 + 16(2x(1-x))^3 - 12(2x(1-x))^2 + 4(2x(1-x))$

(2) Sketch cobweb plots here to answer the following questions.



– – = $X_0 = 0.1$

– – = $X_0 = -0.1$

– – $X_0 = 0.7$

(a) Where are the fixed points of the map?

fixedpts are $P_1 = 0$ $P_2 = 0.5$

(b) If $x_0 = 0.1$, where does the iteration lead? $X_0 = 0.1 \to P_2 = 0.5$
What about $x_0 = -0.1$, $x_0 = 0.9$ $x_0 = 1.1$?

$X_0 = -0.1 \to$ goes to $-\infty$. $X_0 = 0.9 \to$ goes to $P_2 = 0.5$

$X_0 = 1.1 \to$ goes to $-\infty$ (see below for cobweb plot)

(c) Which fixed point is attracting (a sink)? Which one is repelling (a source)?

$P_2 = 0.5$ is an attracting fixed pt.

$P_1 = 0$ is a repelling fixed pt.

(d) Find the basin of attraction for the sink.

$\{x : x \in (0,1)\}$ is the basin of attraction for the fixed pt

(e) Find the set of points that repell from the sink.

$\mathbb{R} / (0,1)$