

Worksheet #13: Motion in a potential field

Consider the differential equation $x'' + 1 - 3x^2 = 0$.

(1) What is $\frac{dP}{dx}$? What is $P(x)$?

(2) Sketch $P(x)$.

(3) Write the differential equation as a first order system.

(4) Find the equilibria.

(5) Sketch the level curves of $E(x, x') = 1/2(x')^2 + P(x)$ in the plane:

(6) What kinds of periodic orbits can happen? What range of energies E may they have?

(7) When is the motion unbounded?

(8) Deduce the stability using the Jacobian Df at the equilibria.