Worksheet #11: Points in the Mandelbrot set

Definition: $M = \{c \in \mathbb{C} : 0 \text{ is not in basin of } \infty \text{ for the map } \mathrm{P_c(z) = z^2 + c}\}$

(1) Is $c = -1$ in M?

(2) Is $c = 1$ in M? How many iterations did you need before you believed this?

(3) Is $c = i$ in M?

(4) Check the *stability* of the orbit you just formed. (i.e., Is it a sink, source or saddle?) [Hint: either use $\begin{bmatrix} x \\ y \end{bmatrix}$ map in $\mathbb{R}^2$ or cheat and use the 1D formula.]

(5) What does Fatou's theorem tell you about if another sink could exist?

(6) So what shape/size is $J(c)$ for $c = i$?

(7) Do you expect $c = i$ to be in the interior, boundary, or exterior of M? [Hint: perturb c]

(8) Find a simple linear conjugacy between $P_c(z) = z^2 + c$, where c, and z are real and the logistic function $g_a(x) = ax(1 - x)$, for some a related to c.