

$$X = \frac{1 - \cos \theta}{2}$$

Plug in

$$G(x) = 4x(1-x) = (2-2\cos\theta) \left(1 - \frac{1}{2} + \frac{1}{2}\cos\theta\right)$$

$$= (2-2\cos\theta) \left(\frac{1}{2} + \frac{1}{2}\cos\theta\right)$$

$$= (1-\cos\theta)(1+\cos\theta) = 1 - \cos^2\theta$$

$$= \frac{1 - \cos 2\theta}{2} \text{ via half angle formula.}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

So, by fixed pt iteration

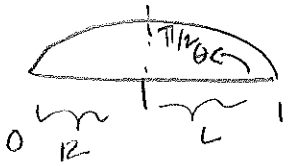
$$y_{n+1} = G(x_n) \Rightarrow \cos(\theta_{n+1}) = \cos(2\theta_n) \left. \begin{array}{l} \text{only way true} \\ \text{statement} \end{array} \right\}$$

$$\Rightarrow \theta_{n+1} = 2\theta_n$$

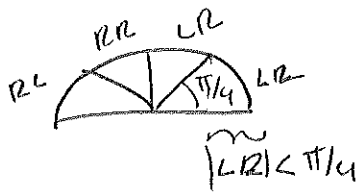
so iterating doubles angle!

What do iterates look like in terms of θ ?

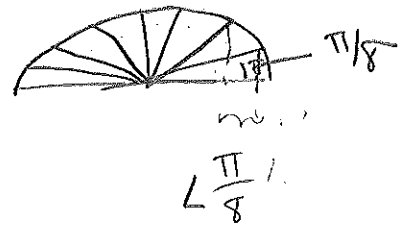
$k=0$



$k=1$



$k=2$



So the length of a subinterval $< \pi/2^k$ on level k .