7/21/06
Alex Barnett

MATH 22: A belated proof!

$\ast$ $\det(AB) = (\det A)(\det B)$

We showed that $\det E$ (elementary matrix) $= \begin{cases} 1, & \text{adding multiple to another row} \\ -1, & \text{swapping} \\ k, & \text{rescale a row.} \end{cases}$

And that these factors are the same as the direct effect of the row op. on det A

So $\det(EA) = (\det E)(\det A)$ for any single row op. $(\dagger)$

Now $\det(AB) = \det(\underbrace{E_p \cdots E_1}_{\text{any invertible } A \text{ can be written as sequence of row ops.}} B) = (\det E_p) \det(E_{p-1} \cdots E_1 B)$ applying $(\dagger)$

$= (\det E_p)(\det E_{p-1}) \cdots (\det E_1)(\det B)$ applying $(\dagger)$ repeatedly.

$= \det(E_p \cdots E_1) \det(B)$ applying converse of $(\dagger)$ repeatedly.

$= (\det A)(\det B)$

However, if A not invertible the above doesn't work, but $\det A = 0$ and AB also not invertible (eg. Midterm 1 question 2 d), so $\det(AB) = 0$ too, and the formula still holds.