Math 53 Chaos!: Homework 2

due Thu Oct 8 ... but best if do relevant questions after each lecture

Chapter 1 is such a lovely part of this book that here you're going to squeeze more out of it! Remember to show your reasoning/working, try and explain proofs as rigorously as you reasonably can.

T1.14 For b) only do RRL (not the other remaining six).

T1.16 In b) there's a typo: for f^6 read G^6. Be careful to explain in each case if $x_0 \neq 1/2$, *why* that must be so from the itinerary.

1.6 [You may use 1/4 apart instead].

1.8 [easy but creative]

Challenge 1: This shows you the power of itineraries. Read p. 32-33 then work through Steps 1-5. Please write up (counting as regular HW questions): Step 3 (answering: which itinerary does y lie in?), and Step 4 (what is the form of the length-$(5k+1)$ sequences, and what is the nearby point that eventually maps d apart?). BONUS: Use Step 4 to explain the correspondence in Step 5.

Lab Visit 1: Read p. 39-42, then adapt your Matlab code from HW1 to model the beetle populations, *i.e.* to numerically evolve Eqs. (1.6) for $N = 200$ iterations. [Hint: make x a 3-by-$(N+1)$ array instead of 1-by-$(N+1)$. Use each row to represent one of the variables L, P, and A. See last part of `intro53.m`].

a) Print out graphs of L_t (larval population) vs t (in time steps) for the two choices $\mu_a = 0.27$ and $\mu_a = 0.96$. If you got your equation correct this will show the same behavior as observed in experiments on p. 41. (You may choose 100 for all starting populations).

b) For $\mu_a = 0.96$ make a 3D scatter (*i.e.* no lines) plot of points $(L_t, P_t, A_t) \in \mathbb{R}^3$, and use it to answer: what kind (dimension, topology?) of attractor in $\mathbb{R}^3$ does the aperiodic-looking orbit settle on to? [Hint: look up `plot3`; also `axis vis3d` will help. You may want to increase N but discard the first few hundred iterations].

c) Describe in 1 sentence how you could test whether this $\mu_a = 0.96$ behavior is *chaotic*. BONUS: Perform this test—is it?

Comp expt 2.1: p. 52. Use my code `explormap2d.m` to mess around with the Hénon map and find the requested a value to 2 decimal places (trial and error? why is it hard?) where the orbit type changes.

2.1 (easy, review of Math 22)

2.3

T2.5 (easy)