

Worksheet #4: Dimensional analysis II

In this worksheet we explore the fundamental solution for the heat equation (without calculus.) A pulse of energy sized e is released at the origin at time $t = 0$. The medium has heat capacity (energy per volume per degree) and thermal conductivity K (power per length per degree). The temperature at distance r and time t is u . (We take $u = 0$ everywhere for $t < 0$.)

- (a) Using the fundamental units energy (E), length (L), time (T), and temperature (Θ), construct the 4×6 dimensional matrix A . (Hint: the fundamental units of K are $EL^{-1}T^{-1}\Theta^{-1}$.)

- (b) Find the $p = 2$ independent dimensionless quantities. Find one that does not involve u , and the other that does not involve r .

- (c) What does the Buckingham Pi theorem tell us about these quantities. Use this to find a function u of the everything else.

- (d) If $r = 0$, how must u scale with t ?

- (e) How does the scaling in part (d) change in a general dimension d . (We had $d = 3$ above. Note, that K has units $ET^{-1}L^{2-d}\Theta^{-1}$ in general.)