Worksheet #19: Convolution and the Fourier Transform

(1) Let u and v be Schwarz functions. Show that
$$\mathcal{F}\left(u * v\right)(\xi) = \hat{u}(\xi)\hat{v}(\xi).$$

(2) Work out the convolution of $u(x)$ and $v(x)$.

(a) $u(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$v(x) = u(x)$

(b) $u(x) =$ any function. $v(x) = \delta(x)$ The delta function.

(c) $u(x) = v(x) = e^{-\frac{x^2}{2}}$ How wide is the answer compared to the original?