

Worksheet #11: L^2 convergence

(1) Is the sequence of functions $f_n(x) = \begin{cases} 1 & x < \frac{1}{n} \\ 0 & \text{otherwise} \end{cases}$ convergent on $(0, 1)$? If so, in what sense? (pointwise, uniformly, or L^2)

(2) Is the sequence of functions $f_n(x) = \begin{cases} \sqrt{n} & x < \frac{1}{n} \\ 0 & \text{otherwise} \end{cases}$ convergent on $(0, 1)$? If so, in what sense? (pointwise, uniformly, or L^2)

(3) Now consider the unbounded interval $(-\infty, \infty)$. Is the sequence of functions $f_n(x) = \begin{cases} \frac{1}{n} & |x| < n \\ 0 & \text{otherwise} \end{cases}$ convergent? If so, in what sense? (pointwise, uniformly, or L^2)

(4) Modify problem (3) so that the function converges pointwise and uniformly but not in L^2 .