## Worksheet \#8: Initial layer

Consider the small mass damped spring equation

$$
\begin{array}{rlrl}
\epsilon y^{\prime \prime}+y^{\prime}+y & =0 & & t>0 \\
y(0)=0 & \epsilon y^{\prime}(0) & =1 & \\
\epsilon \ll 1
\end{array}
$$

(1) Write down and solve for the outer layer. [Hint: take $\epsilon=0$ ] Can you identify the constant?
(2) Rescale the ODE in terms of time taking $\tau=\frac{t}{\delta(\epsilon)}$ and $Y(\tau)=y(t)$.
(3) Use dominant balancing to choose a scale $\delta=\epsilon^{\alpha}$ for some $\alpha$.
(4) Rewrite the ODE with this choice of $\delta$.
(5) Find the leading order equation and solve. If possible, find the constants involved.
(6) Match the two solutions by identifying the constant in part 1 . What is the uniform approximation to the solution?

