Worksheet #8: Initial layer

Consider the small mass damped spring equation

$$\epsilon y'' + y' + y = 0 \qquad t > 0$$

$$y(0) = 0 \quad \epsilon y'(0) = 1 \qquad \epsilon \ll 1$$

(1) Write down and solve for the outer layer. [Hint: take $\epsilon = 0$] Can you identify the constant?

(2) Rescale the ODE in terms of time taking $\tau = \frac{t}{\delta(\epsilon)}$ and $Y(\tau) = y(t)$.

- (3) Use dominant balancing to choose a scale $\delta = \epsilon^{\alpha}$ for some α .
- (4) Rewrite the ODE with this choice of δ .
- (5) Find the leading order equation and solve. If possible, find the constants involved.

(6) Match the two solutions by identifying the constant in part 1. What is the uniform approximation to the solution?