Worksheet #3: ODE review

(1) Show that the transformation $w = u^{1-n}$ makes the "Bernoulli equation"

$$u'(t) + p(t)u(t) = q(t)u^n(t)$$

(which looks nonlinear) into a linear equation. In other words, equation is of the form $v'(t) + \tilde{p}(t)v(t) = \tilde{q}(t)$. What are the functions $\tilde{p}(t)$ and $\tilde{q}(t)$?

$$w = u^{1-n} \Rightarrow u = w^{\frac{1}{1-n}} \qquad u' = \frac{1}{1-n} w^{1-\frac{1}{1-n}} w' = \frac{w^{n/1-n} w'}{1-n}$$

So $\dfrac{w^{n/1-n} w'}{1-n} + p(t) w^{1/1-n} = q\, w^{n/1-n}$

multiplying by $\dfrac{1-n}{w^{n/1-n}}$ $\Rightarrow$ $w' + \underbrace{(1-n)p(t)}_{\tilde{p}(t)} w = \underbrace{(1-n)q(t)}_{\tilde{q}(t)}$

(2) What method(s) would you use to solve the following ordinary differential equations? Note you may need more than one.

(a) $u'' + 2t\,(u')^2 = 0$

let $v = u'$

Then $v' + 2t\, v^2 = 0$

This equation is separable.

(b) $u'' + 3u' + 2u = t$

Use constant coefficient solution $u = e^{rt}$

roots are $r_1 = -2$, $r_1 = -1$.

+ variation of parameter to find particular soln.

(c) $u'' + u' = u + \ln t$

Find homogeneous soln via constant coefficient 2nd order solver

then use variation of parameter.

(d) $\dfrac{u'}{u} = t^2u^3 + \dfrac{1}{t}$

Use problem 1

+ integrating factor.