Worksheet #3: ODE review

(1) Show that the transformation $w = u^{1-n}$ makes the "Bernoulli equation"

$$u'(t) + p(t)u(t) = q(t)u^n(t)$$

(which looks nonlinear) into a linear equation. In other words, equation is of the form $v'(t) + \tilde{p}(t)v(t) = \tilde{q}(t)$. What are the functions $\tilde{p}(t)$ and $\tilde{q}(t)$?

(2) What method(s) would you use to solve the following ordinary differential equations? Note you may need more than one.
(a) u'' + 2t (u')² = 0

(b) u'' + 3u' + 2u = t

(c)
$$u'' + u' = u + \ln t$$

(d)
$$\frac{u'}{u} = t^2 u^3 + \frac{1}{t}$$