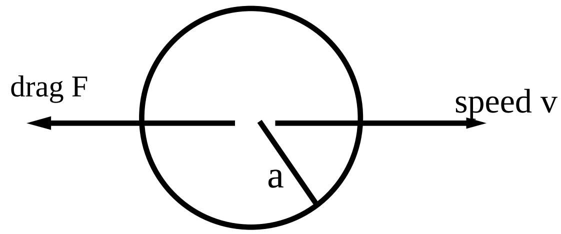
Worksheet #1: Dimensional Analysis

Say we suspect that drag force F depends only on a sphere's radius a, its speed v, and the surrounding fluid density ρ.



a) What are the dimensions of a, v, ρ and F?
Solution:

$$[a] = L$$
$$[v] = LT^{-1}$$
$$[\rho] = ML^{-3}$$
$$[F] = MLT^{-2} \qquad F = \text{mass} \times \text{acceleration}$$

b) Create the dimensions matrix for this problem.
Solution:

	a	v	ρ	F
M	0	0	1	1
L	1	1	−3	1
T	0	−1	0	−2

c) Find a dimensionless combination of the quantities, π.
Solution: There are many.
$\pi = \frac{F}{v^2\rho a^2}$ or $\pi = \frac{v^2\rho a^2}{F}$ or $\pi = \left(\frac{F}{v^2\rho a^2}\right)^k$ for any k.

d) Find $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]$ so that $\pi = a^{\alpha_1} v^{\alpha_2} \rho^{\alpha_3} F^{\alpha_4}$. Is this choice unique? Find the subspace of all such vectors and find a basis.
Solution: $\boldsymbol{\alpha}_1 = [-2, -2, -1, 1]$ is not a unique choice of $\boldsymbol{\alpha}$. Any other choice of $\boldsymbol{\alpha}$ will satisfy $\boldsymbol{\alpha} = k\boldsymbol{\alpha}_1$.

e) What is the number of independent dimensionless parameters?
Solution: There is only one independent dimensionless parameter since the rank of the 3×4 A is 3 and $\dim(\text{Null}(\mathsf{A})) = 4\text{-rank}(\mathsf{A}) = 1$.

f) What does the Pi Theorem tell us for this problem? How must F depend on a, v, ρ?
Solution: The Pi Theorem tells us that the π is constant. In other words, $\pi = \frac{F}{v^2\rho a^2} = c$ for some constant c. So $F = cv^2\rho a^2$.

g) If F also depended on visocity η (units $ML^{-1}T^{-1}$) Repeat part e). **Solution:** Repeat the process. You will find there two dimensionless quantities. π_1 defined as π from before and $\pi_2 = \frac{\eta}{\rho v a}$ is the physical quantity known as the Reynold's number. The function F in terms of the other quantities is $F = cv^2\rho a^2 g(\pi_2)$ where g is some function.