Worksheet #15: Image deblurring (1D)

Consider the symmetric blurring operator $Kf(x) = \int_{-\pi}^{\pi} k(x-y)f(y)dy$, where $k(s)$ is even, symmetric, and 2π-periodic. $k(s)$ is called an aperture function.

(1) Show that $\phi_n(x) = 1$ is an eigenfunction of K, and find its eigenvalue? [Hint: why is $K\phi_n(x)$ independent of x? Why is λ_0 independent of x?]

(2) Show that $\phi_n(x) = \cos(nx)$, $n = 1, 2, \ldots$ is an eigenfunction of K, find its eigenvalue of λ_n. [Hint: use addition formula, k even]

(3) How do λ_n relate to Fourier cos coefficients K_n of aperture function $k(s)$?

You could check that $\sin(nx)$ is also eigenfunction with same eigenvalue λ_n. Assume image is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$ and $K(x) = g(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(nx) + B_n \sin(nx)]$

(4) How are g's Fourier coefficients related to those of f?

Such is the nature of convolution kernels. How would you invert $g \to f$ ie. deconvolve?